

IDEALIZATION III:
APPROXIMATION
AND TRUTH

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WHY DO IDEALIZATIONAL STATEMENTS APPLY TO REALITY?

I. Introduction

The problem of how the language hooks up onto the world is as old as the world, or at least as old as the language. In contemporary philosophy of science, it reappears as the problem of how a theory applies to reality. It may be at first supposed that this is just a special case of the old epistemological query. That it is not becomes evident when we reflect on the fact that profoundly false (in the correspondence sense) theories somehow manage to apply to reality, or at least we have a tendency to think that they do. Who would deny that Newton's theory of gravity applies to objects in our world? Yet, it appears funny to even talk about truth or falsity as Newton's law does not even talk about reality. Instead, it has pretensions to be true only for isolated pairs of material points.

Below we propose a way of understanding the notion of application in terms of the Idealizational Conception of Science. We shall also suggest that L. Nowak's handling of this issue is unsatisfactory.

II. An Ambiguity in the Concept of Application

I. The peculiar nature of idealizational theories and statements indicates an ambiguity in the concept of application. Let us talk about idealizational statements first, and extend the concept to cover idealizational theories later.

There is a sense in which an idealizational statement applies only to abstract models. (This is the sense with which some version of the concept of correspondence-truth is tightly associated.) We will say that the statement *holds for* the model. There is yet another sense (there better be) in which an idealizational statement applies to reality. We will

take this to be the proper sense of application as it incorporates the strong practical connotation of the term. We will thus adopt the convention of speaking of a statement *applying* to reality only in this sense. Thus, Newton's law of gravity holds for isolated pairs of material points but it applies to all (pairs of) physical objects. Surely enough, the distinction is of no use unless further explained.

2. The received conception of application, as we will see below, identifies the two concepts. Scientific statements apply to reality *because* they hold for reality.

Although logical positivism has been abandoned as a research program many of its (pre)conceptions persists for better or for worse. This is evident in Nancy Cartwright's account [cf. her 1983]. Her stance is one of the most forceful pleas to recognize the role of idealization in contemporary anglosaxon philosophy of science. She recognizes the fact that science does not follow the path envisaged by logical positivists: it does not use only phenomenological laws (Cartwright's term for factual laws), in fact, these are quite rare. It makes use of theoretical laws (her term for idealizational laws) that are, strictly speaking, false. They do not hold for reality and so Cartright following logical positivists concludes that the fundamental laws of science do not apply to reality.¹

This conclusion seems unsatisfactory though it is inevitable on the Received account of application. Whatever the general merits of this strategy we shall here attempt to account for the intuition that idealizational laws do apply to reality by rejecting the positivist view.

III. The Realistic Assumption

Let us now consider how the Idealizational Conception of Science fares with respect to the question of application. Before we do so let us briefly rehearse the basic assumptions.

1. A theory on this view is a sequence of idealizational statements whose antecedents are constituted by a sequence of idealizing assumptions, where the secondary factors distinguished by a theory are assumed to adopt extreme values (often zero); and whose consequents express a relation between the investigated factor and those factors which are not idealized in the antecedent [Nowak 1980]. Its general form can be thus represented as:

- (1) if $P(x) \neq 0$ & $p_1(x) \neq 0$ & ... & $p_{k-1}(x) \neq 0$
 & $p_k(x) = 0$ & ... & $p_n(x) = 0$
 then $F(x) = g_k(P(x), p_1(x), \dots, p_{k-1}(x))$

where P is the principal factor, and p_1, \dots, p_n are the secondary (or, disturbing) factors; g_k is the function relating the nonidealized factors of (1), viz. P, p_1, \dots, p_{k-1} , to the investigated factor F .

Let us use a simple illustration. Galileo's² theory of free falling bodies distinguishes two factors relevant to falling bodies: {mass (m), air resistance (R)}. Subsequently, he idealized from the latter, considering a counterfactual: 'what would happen were there no air resistance?'. This made him postulate the following hypothesis, which need be expressed in the form of an idealizational statement:

- (2) If $m \neq 0$ & $R = 0$ then $F = gm$

Subsequently, the idealizing assumption was removed, i.e. the first idealizational statement was concretized, so as to obtain a factual statement (one with no idealizing assumptions):

- (3) If $m \neq 0$ & $R \neq 0$ then $F = gm - R$

Galilean theory, on this account, is the sequence, <(2), (3)>.

Nowak faces a dilemma now. If he uses the positivistic notion of application, idealizational statements will have a very limited domain of application (on an enriched ontology) or will not apply at all (on an ordinary ontology). He posits that the general form of the idealizational statement (1) considered above is incomplete. It needs to be supplemented by another term in the antecedent of the conditional, the so-called realistic assumption:³

- (1*) if $G(x)$ & $P(x) \neq 0$ & $p_1(x) \neq 0$ & ... & $p_{k-1}(x) \neq 0$
 & $p_k(x) = 0$ & ... & $p_n(x) = 0$
 then $F(x) = g_k(P(x), p_1(x), \dots, p_{k-1}(x))$

It is never stated clearly that the realistic assumption is to solve the problem of the applicability of idealizational statements and for good reason because it does not solve the problem. It is clear, however, that Nowak intended it to play this role. The realistic assumption is defined as a "propositional function $G(x)$ which is fulfilled by any element of the universe [of discourse] U ." [1980, p. 28] It is evident thus that the extension of the realistic assumption is the domain of *application* (or *the intended model*).

We should ask ourselves, however, how satisfactory this solution is. Why do idealizational statements apply to reality? The answer here given is: because each idealizational statement contains a propositional function delimiting its domain of application. To wit, idealizational statements apply to reality because they say they do.

2. There is yet another reason not to feel satisfied with this solution. It is instructive to see what kinds of realistic assumptions are given in actual reconstructions of idealizational theories. The realistic assumption for the Marxian theory of value as reconstructed by Nowak [1980] is “ x is a commodity.” Nowakowa, following Such’s [1972] reconstruction, interprets the realistic assumption of Galileo’s theory as a conjunction of two propositional functions: “ x is a body” and “ x is falling”.

It ought to be noticed that as a rule the realistic assumptions are ordinary language terms serving as intuitive delimitations of the domain in question. Being a ‘body’ or ‘falling’ seem peculiar to be *scientific* concepts. A worried philosopher may then ask: ‘But what is a body?’. Are atoms to count as bodies? What about electrons and quarks? Or, for that matter, “falling”? Falling ordinarily occurs in time. Which time interval do we then choose the theory to apply to? At the time of Galileo’s life? But surely, bodies are not always falling. These questions may seem “fishy”, to use Wittgenstein’s term, yet there is nothing in the formula to prevent them from being asked.

Furthermore, there is a sense in which Galileo’s theory holds for bodies that are not falling in any intuitive sense. Consider an imaginary case where the body is ‘falling’ (attracted by the Earth’s gravity) yet it is ‘falling’ in a medium whose resistance is equal to that of the force of gravity. Looking at the body we would say that it is not falling: it is suspended. Yet, Galileo’s theory describes this occurrence perfectly.

What this counterexample demonstrates is merely that the kinds of terms used in realistic assumptions do not suit their purpose. We should not be surprised. As ordinary language terms, they are intrinsically ambiguous.

3. Obviously, there is a sense in which Galileo’s theory applies to falling bodies. But this is not because Galileo intended it to do so. (Or, what amounts to the same thing, because his laws are properly reconstructed as including a realistic assumption to this effect in their antecedents.) He may have indeed not intended his theory of inclined planes to apply to falling bodies, yet it does. Newton may have not intended his theory to apply to celestial movements, yet it does.

The answer to the question in virtue of what a theory applies to real phenomena is facilitated by asking what (aspects) of the real phenomena a theory considers. The answer to this latter question is given by Nowak immediately: the theory considers a space of factors essential to the phenomenon in question. Furthermore a phenomenon is *described* (not explained) by listing the values that the factors adopt. We thus describe a phenomenon of a pencil falling down (on Galileo's theory) if we specify the mass of the pencil, the air resistance acting on it, as well as the force of attraction attached to it.

Galileo's theory applies to the phenomenon of the pencil falling insofar as it is able to consider this phenomenon, i.e. first describe it, and then explain it. This is, however, to say that the theory applies to a phenomenon because it considers as essential those factors that describe it. Consequently, the realistic assumption has to be equivalent to the conjunction of the factors considered essential for the phenomenon in question. In Galileo's case, the realistic assumption "x is a body" and "x is falling" is equivalent to the conjunction of "x has a mass m ", "air resistance R acts on x " and "force F is attached to x ".

In general, then, if the form of the idealizational statement is (1*), as Nowak claims it is, it will be the case that:

(4) ' $G(x)$ ' is equivalent to ' $P(x) \ \& \ p_1(x) \ \& \ \dots \ \& \ p_n(x) \ \& \ F(x)$ '

In fact, we would be inclined to say that $G(x)$ is *explicated* in (4). That is to say, the scientific terms of the conjunction explain what we mean when we speak intuitively about a falling body, or a commodity.

4. Let us now see what happens when (4) is false, and to do so we will consider the sets of objects defined by the predicates in question. The extension of G will be called the *intended domain* of a theory, whereas the extension of the conjunction of factors will be called the *real domain* of a theory. There are four imaginable cases, where the intended and real domains of a theory are not identical. Let us briefly consider each.⁴

A. The intended domain is a proper subset of the real domain. That is there are objects in the real domain which are not to be found in the intended domain.

AA. Such a situation may be relatively innocuous, and we have indeed mentioned it above. The very simplest case is illustrated thus. Assume we want to explain how objects slide down planes, and this will be marked in the realistic assumption, by a conjunction of e.g. 'x is a body' and 'x slides down a plane'. Thus the intended domain consists of bodies sliding down planes. The following conjunction of factors considered by the theory delimits its real domain: x has a mass m and friction f acts on x and

x moves at a certain angle α to the earth and the gravitational force F is attached to x . The case that is included in the latter but not the former is of course the case of falling bodies, where $\alpha = 90^\circ$, and air resistance is identified as a force of friction. This case is not included in the intended domain for there is no clear sense, except for the one that *the theory* dictates, according to which a body falling down to earth be considered a body sliding down a plane. But what a theory 'dictates' belongs to the *real* and not the intended domain.

This situation will, in fact, occur persistently because of the tendency of dichotomization in the ordinary language largely overcome in science. Thus, instead of our two 'notions' of moving and not-moving scientists will talk about 'velocitizing other than zero' and 'velocitizing equal to zero', to use neologisms. Or, instead of talking about bodies that are falling and those that are not falling, Galileo will talk about bodies that are falling, except that in the latter case air resistance (or for that matter, the resistance of a different medium) is as great as to prevent the body from, what we term, falling.

AB. A subcase of this situation may be less innocuous. The above example is generous for it assumes that the theory actually holds for, i.e. is true for, all the cases that it applies to. If this were not the case, however, the mismatch between the intended domain and the real domain becomes a tool of saving theories *ad hoc*. If a hypothesis we propose turns out to hold for a limited domain only we simply announce this as the intended domain and thus save our theory. Is the Oedipus complex not universal? Let's then restrict it to the set of white males of well-to-do classes of the Victorian culture, to take a fictitious example.

Accordingly, whereas the former situation is methodologically irrelevant (although it may be relevant for some purposes, e.g. to a historian of science), the latter is methodologically bankrupt.

B. The real domain is a proper subset of the intended domain. There are objects in the intended domain which are not present in the real domain. In other words, the theory is designed to do more than it can. Galileo sets out to explain and provide a theory of all falling bodies, yet it turns out he only gives a theory that holds approximately for a subset of them, viz. for those that fall near the surface of the Earth.

But, of course, the case is troublesome only because of the presence of the intended domain. Had there not been such, the problem would not arise, the theory would apply just to those objects that it really applies to.

C. The real domain and the intended domain cross-sect properly (this is a combination of the above two cases). Both of the above points hold for this case. We can be happy that there are more objects that we thought the theory would apply to, and disappointed that some of those

we thought the theory would take care of, can not in fact be taken into account by the theory.

D. The real domain and the intended domain are disjoint, i.e. there are no objects in common. A theoretically possible case, where the theoretician misses the point of his theory. Even though, it may seem entirely unsound to include this case here, it nonetheless points to an important question in the light of what has been said above: the question why we need the intended domain at all. For as we tried to show, it does not affect anything, except negatively (consider point **AB** above), and only creates problems which in the end have to be quasi-problems (**AA, B, C, D**).

In all of the above cases the intended domain is spurious. Indeed, mostly so in the fifth possible case when the intended domain and the real domain are identical!

5. Let us now return and try characterize the difference between holding and application. We will follow ordinary usage in saying that the propositional function $P(x) = v$ (where P is a parameter and v is its value for an object x) is *satisfied* by all objects that can be described by means of the parameter and for which the parameter adopts value v . We will say that the propositional function $P(x) = v$ *indicates* all objects that can be described by means of parameter P , but for which the parameter does not necessarily adopt value v . Idealizational statements can thus be said to hold for those objects that satisfy the conjunction of conditions in the antecedent. On the other hand, idealizational statements apply to those objects that are indicated by the conjunction of conditions in the antecedent (with the exclusion of the realistic condition, of course).⁵

IV. Models

1. The word has it that idealizational statements hold for models. This claim is ambiguous, however. This ambiguity is well illustrated by asking how much the body for which Galileo's law of free fall holds weighs. Or, we could ask, for that matter, what the mass of the material points of an ideal gas is. The answer one is tempted to give is quite illuminating: it does not matter. The body can weigh *anything*, the material points of an ideal gas may have *any* mass at all.

We have said that the antecedent of an idealizational statement defines the objects the statement holds for. But there are two senses of 'define'. Consider the simplest example, Galileo's law of free fall, (2). Its antecedent is comprised by a conjunction:

$$(6) \quad m \neq 0 \ \& \ R = 0$$

How many objects have thus been defined? There are two answers. One is that there is an infinity of objects thus defined. Air resistance equal to zero is attached to all of them. They all have mass though its specific value is different: some will have mass equal to 2 g, 145 g, 467773 g, etc.

However, it is also possible to say that only one object has thus been defined: no air resistance is attached to it and it has mass other than zero – but neither does its mass equal 2 g nor 145 g nor 467773 g, nor . . . The one object has an *arbitrary* mass. Indeed, if we inspect (6) more closely we will see that despite our extensionalist intuitions the information that the conjunction provides suffices only for a description of an arbitrary object. If the object were not arbitrary it would be meaningful to ask what mass it has. Is its mass equal to 2 g? The answer is negative.⁶ It is not equal to 145 g, or 467773 g, or any other value either. Its mass is arbitrary.

We shall refer to the set of entities defined (in the weak sense) by the antecedent of an idealizational statement as the counterfactual or ideal domain of that statement. Whereas the one individual defined (in the strong sense) by the antecedent of an idealizational statement deserves the name of a model of the statement.⁷

2. We have claimed that idealizational statements hold for ideal domains not for models. A more interesting claim about the relation between models, idealizational statements and application awaits a more appropriate analysis of arbitrary individuals.

It is standard to analyze arbitrary individuals in terms of classes. (So, when mathematicians or scientists talk about *any* numbers, points, bodies or other entities, what they mean is classes thereof.) If such an analysis is tenable our distinction resolves into a quibble. Below, we shall merely indicate that there are important differences between models (arbitrary objects) and ideal domains (“corresponding” sets of objects).

The worry underlying the characterization of models in terms of arbitrary individuals is understandable. We seem to be explaining the obscure in terms of the even more obscure. It is not clear, however, that the explication standardly proposed is of much help on this score. There is a considerable dissatisfaction with the notion of a set in the literature. The main objection concerns the intelligibility of the “unification” that is somehow supposed to be effected in sets. Even if — so the line of argumentation goes — one could make sense of the set of all books being somehow one (unifying its many members), how does a unit-set “unify” its only member? Or, for that matter, a null-set?⁸

Moreover, it does not seem very reasonable to suppose that the scientist drawing a point on the blackboard and naming it ‘*m*’ means (in both

logical and psychological sense) to represent an infinity of point-masses. What he shows is that what holds of the model also holds of an infinity of point-masses (i.e., for the whole ideal domain). This is the kind of reasoning that underlies the standard rule of universal instantiation in general. However, were we to equate the arbitrary model-individual with the class of individuals from the ideal domain, and were we to hold that what holds of that special individual holds of each and every individual we would be committing a category mistake. We would be inferring from the fact that something holds of a class that it holds of each member.⁹

3. To every idealizational statement then there corresponds a model and a domain. Corresponding to two extreme statements, there is an ideal model and an ideal domain defined (in the respective senses) by the antecedent of the idealizational law. At the other end, there is the concrete model and reality.

V. Why do Idealizational Statements Apply to Reality?

1. The positivist answer to this question which has persisted in many disguises was that scientific statements apply to reality because they hold for reality. We know that most of the statements do not hold for reality. It follows that either we must change our conception of why idealizational statements apply to reality or else we must follow Cartwright in claiming that they do not.

Given our above considerations we are committed to saying that idealizational statements apply to reality *because* they hold for ideal domains. Below, we will explain what this amounts to.

2. The reason why the positivistic account of application is so captivating is that the notion of application is closely tied to that of generality. *Prima facie*, the more general a theory is the greater is its applicability.

It must be noticed, however, that there are at least two concepts of generality. The concept of generality stemming from the Aristotelian representationalist tradition¹⁰ is the logical notion of universality as exemplified by the universal quantifier. Accordingly, a universally generalized statement $(x)P(x)$ is true to the extent that the propositional function $P(x)$ is satisfied by all objects in the domain of discourse. In this sense, Newton's law of gravitation is general to the extent that it holds for all pairs of physical bodies.

The other notion of generality stems from the Plato-Hegelian deformationalist tradition. To wit, an (idealizational) statement is general to the extent that it holds for *no* objects in the domain. To be more precise,

an (idealizational) statement is general to the extent that it holds for some objects that are idealizations of *all* objects in the domain of discourse. In this sense, Newton's law is general because it holds for isolated pairs of material points which are idealizations of all pairs of real objects.

We thus see that the deformationalist notion of generality is not entirely counterintuitive. However, the 'all'-generality is indirect, so to speak. The domain of all objects is involved in a relation to the objects for which the law holds — it is not related to the law directly. The relation of application is thus composed of two relations: the relation of holding-for and the relation of being-an-idealization-of.

The relata in the relation of application are scientific statements (idealizational among them) and *all* objects. The mistake of the positivists was that they took the relation of application to be identical with that of holding for (for which the rationale is given by their logical conception of generality). That it was a mistake becomes evident when seeing that the relation of holding does not obtain between idealizational statements and all relevant objects. It holds between idealizational statements and ideal objects. There is yet another relation involved, that of being an idealization that holds between ideal objects and *all* objects (in the domain of discourse). The relation of application is thus composed of the relation of holding for (obtaining between scientific statements and ideal objects) and the relation of being an idealization of (obtaining between ideal objects and *all* objects). The positivist thesis identifying the relation of application with that of holding for is true in the special case of factual statements.

We can thus conclude that an idealizational statement applies to reality because it holds for an ideal domain, and because objects in the ideal domain are idealizations of objects in reality.

VI. Final Remarks

1. We have argued that the notion of a realistic assumption is spurious and that it denotes the conjunction of factors considered essential by a theory. We have also shown that the positivist view of application was incomplete. They identified the concept of application with the concept of holding. The former seems better reconstrued if supplemented by another idealizational component.

2. Let us close by extending the concept of application to encompass idealizational theories, defined as sequences of idealizational statement of decreasing abstractness up to the factual statement. We will say that

a simple idealizational theory holds for the sequence of the ideal domains corresponding to the sequence of idealizational statements comprising it. It will be said to apply to reality, on the other hand, to the extent that it holds for the sequence of ideal domains and the objects in the ideal domains are idealizations of the objects in the reality.

A simple idealizational theory T applies to an object x iff there is a sequence of domains D_1, \dots, D_k corresponding to the sequence of idealizational statements I_1, \dots, I_k of T and x belongs to D_k . (Since I_k need not be a factual statement, x does not have to belong to reality.) In the special case, when x belongs to reality, this amounts to saying that the theory applies to x just in case all its idealizational statements apply to x .

What may be now called the proper domain of a simple idealizational theory is neither its intended domain nor reality, it is rather the sequence of domains the first of which is the ideal domain of a theory and the last of which is the reality. A model of a theory (as opposed to a model of a statement) is the sequence of models, of which the ideal model is the first, and the concrete model is the last.

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NOTES

¹ "My basic view is that fundamental equations do not govern objects in reality; they govern only objects in models." [Cartwright 1983, p. 129]. Cartwright [1989] has recently modified her view on this issue. She has suggested that we think of the idealizational statements as talking about nature's capacities – causal powers underlying the complex patterns of observable reality.

² For the sake of simplicity, we take the Newton-reinterpreted version of the theory of free fall.

³ It should not be supposed that this is how Nowak introduces the realistic assumption. He does not first introduce (1) only to be followed by (1*). We have reconstructed the sort of reasoning that must have underlaid the introduction of the realistic assumption to better understand why it is indeed unsatisfactory with respect to the only function that it can have.

⁴ These partially overlap with Nowak's [1979] discussion of the adequacy of theories.

⁵ There is a possibility that must be mentioned here. It may very well be the case that a theory is formulated for an explicitly limited domain, say $P(x) = v$, where this is not an idealizing assumption. According to our claim the theory will hold for those objects that satisfy this condition (and possibly others) but it will apply to all objects indicated by this condition (and others), i.e. precisely to those objects that the theory was designed to avoid.

It might be that the distinction between idealizing and non-idealizing conditions could be helpful here. We could then restrict our claim that the theory applies to all objects indicated by only the idealizing conditions and satisfied by non-idealizing conditions. However, there does not seem to be a very solid grasp on such a distinction. On the other hand, one could bite the bullet and maintain that such restrictions are not captured by the account because they are *ad hoc*. Those restrictions must be deemed temporary and efforts should be made to find a reducing or corresponding theory.

⁶ Given the infinite number of possible values, the probability that the object has the mass equal to any given value is 0.

⁷ This is a different notion of scientific modelling from that considered by Nowak [1980]. The notion there employed is based on the notion of (formal) analogy. Here we are concerned with a different sense of the concept, viz. when scientists create 'abstract models' designed to consider a phenomenon in question. The former concept relates two theories: one theory is modelled on another with respect to an analogy, the latter occurs within a theory. Moreover, in his [1987] paper, Nowak purports to homogenize the apparently inhomogeneous concept of a model, yet it seems that neither of the four senses he distinguishes comes close enough to the one proposed here (the second sense may seem to be related, however).

⁸ If we were to think of sets as mere collections of their members (excluding the troublesome feature of unification somehow) the case would not be aided. The explication of models in terms of thus conceived classes would simply be inadequate. A scientific model (of a law or theory) is in an important sense one and not infinitely many.

⁹ There is a sense in which the relation between the arbitrary object and all individuals from the ideal domain is at least analogous to the relation between ideal and real objects. It is not clear, however, that it is the same. Whether idealization could illuminate the concept of arbitrary objects deserves separate attention.

¹⁰ For the distinction of the two traditions, cf. Nowak [1980]. An explication of the deformationalist notion of truth for idealizational statements is presented by Nowakowa [1991].

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