IDEALIZATION I: GENERAL PROBLEMS

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REDUCTION AND CORRESPONDENCE IN THE IDEALIZATIONAL CONCEPTION OF SCIENCE

The notion of reduction and correspondence has been very often thought to be the same in many theories of science. Both have been supposed to function as motors of the development of science. I. Nowakowa, in her article [1974] has shown that the reconstruction of the notion of correspondence in terms of Idealizational Conception of Science, which she argues is superior to the other notions, allows one to show that there is an important difference between the two. In fact, she argues, it is dialectical correspondence and not reduction which is suited to constitute the principle of the development of science.

The reservation that one could express at the very beginning is that although I. Nowakowa carried out the reconstruction of the notion of correspondence, she did not do so with respect to the notion of reduction. The main aim of this paper, the reconstruction of the notion of reduction in terms of idealizational conception of science[1], may thus also be seen as a reconsideration of the relation between the notion of correspondence and reduction.

1. Dialectical correspondence

The notion of dialectical correspondence provides a means of relating laws of two different theories. The definition of dialectical correspondence given by I. Nowakowa is the following. An idealizational law *T'* dialectically corresponds to an (idealizational or factual) law *T* iff:

1. *T* is formulated at an earlier stage of the development of knowledge than *T'*,
2. *T* has the same consequent as the consequent of a logical consequence *cT'* of *T'* (in particular *T'* itself),

3. the realistic assumptions of T are identical to the realistic assumptions of cT',

4. the sequence of idealizational assumptions of T is a proper subsequence of the sequence of idealizational assumptions of T',

5. it is possible to derive a concretization T'' of T' with identical idealizational assumptions as in T.

Dialectical correspondence is thus a principle lying behind the development of science. The progress thus envisaged consists in the discovery of factors which the previous theory has not encountered. But only in the discovery of additional secondary factors. Thus it is claimed the old theory is modified.

Dialectical correspondence cannot accordingly account for all the phenomena of development of science. It cannot account for the extension of the domain of application of a theory (condition 3). Hence, Newton's theory does not correspond to Galileo's theory. The relation of these two theories is often termed that of reduction. Let us thus investigate what such a relation would consist in.

2. *Dialectical reduction*

In the positivist framework, the ("special case") reductions have been thought to be unproblematic, the relation of reduction being really the relation of logical derivation. The inadequacies of this view have been pointed out by, among others, Feyerabend [1985] and Kuhn [1962].

There are thus two problems that any account of reduction (of the first kind to which we will limit ourselves here) has to address. Firstly, it must specify the relation between the reducing and the reduced theory, and specifically it must account for the "derivability" of laws of the one theory from another. Secondly, it must provide an account of what happens with the meanings of the terms of the reduced theory. The last problem will not, however, being discussed in this paper.

The core of the first problem lies in the fact that the reducing theory T makes contradicts the claims of the reduced theory t. Taking the example of the Galileo-Newton reduction, it is seen that whereas for the former the "gravitation" is a constant, for the latter it changes with the ratio of the height H above the ground and the radius R of the Earth. The claim thus is that although both theories yield comparable, indeed approximately the same results within a certain boundary conditions, this does not suffice for *logical* derivation.

The "contradiction" consists in the fact that what the reduced theory takes to be a constant (g) is considered to be a variable by the reducing theory (GM/r^2). That is to say the reduced theory does not take into account these variables. Indeed this is the very fact which is being of a more restricted application (for some values of M and r, g will be identical with GM/r^2).

The other important fact which allows for their applicability to a class of the same phenomena is the fact that although the reducing theory has more variables (those which allow to interpret the constants of the reduced theory) it nonetheless has some of the variables in common. In the above case it will of course be the mass of a falling body. Such a common set of factors will be termed the conservative space.

The reducing theory T gives thus a deeper insight into the phenomenon, it postulates another factor which not only has not been dicovered yet (for that could be the case with many factors[2]), but which is more fundamental (= essential) than any other of the thus far discerned factors. There is still a further sense in which the newly postulated factor, and consequently the whole theory, is more fundamental than the old one, *viz.* the sense in which the laws of the reduced theory are "derivable" from the laws of the reducing one.

In terms of the idealizational conception of science one could express this in the following way. The space of factors of the reducing theory T, has a greater cardinality than that of the reduced theory t. In fact the "additional" (with respect to t) factors in T are more essential than those that the two theories have in common (conservative space). It is the difference in essentiality which allows one theory (t) to be applicable only to a subset of the phenomena to which the other (T) is applicable.

DEFINITION. Let theory t and theory T be characterized by the following sequence of idealizational laws:

[t_1] if $U(t)x$ & $P(x) \neq 0$ & $Q(x) = 0$, then $F(x) = k(P(x))$
[t_2] if $U(t)x$ & $P(x) \neq 0$ & $Q(x) \neq 0$, then $F(x) = k'(P(x), Q(x))$

[T_1] if $U(T)x$ & $R(x) \neq 0$ & $P(x) = 0$ & $Q(x) = 0$, then $F(x) = g(R(x))$
[T_2] if $U(T)x$ & $R(x) \neq 0$ & $P(x) \neq 0$ & $Q(x) = 0$, then $F(x) = g'(R(x), P(x))$

[T_3] if $U(T)x$ & $R(x) \neq 0$ & $P(x) \neq 0$ & $Q(x) \neq 0$, then $F(x) = g''(R(x), P(x), Q(x))$

A theory t is reduced to another theory T, iff:
(I) the domain of t is a subset of the domain of T;
(II) the investigated parameter (F) is the same for t and T;
(III) there is a common set (the conservative space), C, of the space of factors essential to F in t and the space of factors essential to F in T;
(IV) the most essential factor of T does not belong to C.

Condition (I) is justified by the fact that we would want the reducing theory to be of greater scope than the reduced theory. The reducing theory must explain more phenomena than the reduced one. Condition (II) merely postulates that the two theories should be trying to explain the very same thing, e.g. the behavior of falling bodies. This does not prevent them from explaining more phenomena. The reason being that many phenomena might be having the same factors essential to them. Thus Newton's laws of gravitation explain both – the phenomena of celestial mechanics, for they consider enough parameters to do so, in particular the two (attracting) masses, as well as the phenomena of free falling bodies, for again two masses and distance are involved. Condition (III) demands that there be factors in common as used by one and the other theory. This secures the continuity between the theories. Otherwise, one might simply acclaim them utterly incompatible. Condition (IV) is a condition of reductive superiority of one theory (the reducing theory) over the other (the reduced theory). It assures that the most essential factor of T is unique to T.

It will be noticed that there are three main differences between the dialectical correspondence and reduction. Firstly, whereas dialectical correspondence between theories consists in the fact that the later theory considers more secondary factors than the earlier one, the reducing theory proposes different principal factors, and takes all the factors (principal and secondary) of the reduced theory as its secondary factors (in the simplest case retaining the essentialist ordering, cf. simple vs. complex reductions).

As a consequence of this, the domain of the reduced theory (defined by the realistic condition) is a subset of that of the reducing theory, whereas in the case of correspondence the domains of the theories are the same.

Thirdly, reduction is not defined with respect to time. It is crucial for correspondence that the later theory be the one which contains more secondary factors than the earlier. The case with reduction is somewhat different. For although there are cases of reduction where one theory becomes "subsumed" by another, as the discussed case of Galileo-Newton reduction, there are cases of non-subsumptive reductions (characteristic of interdisciplinary theory relations) where only part of the reduced theory becomes "subsumed" by the reducing theory.

DERIVABILITY. The reducing theory postulates a more essential factor than those postulated by the reduced theory in explaining a phenomenon. In t, R does not belong to the space of factors essential to F, whereas it does in T. The difference is substantial, for it involves the ontic commitment of the two theories. Whereas T postulates three factors to be essential to the investigated parameter, *viz.* $\langle R, P, Q\rangle$, t postulates only two $\langle P, Q\rangle$. In an alternative notation $S/F(t) = \langle {}^{-}R, P, Q\rangle$, where "${}^{-}R$" means "$R$ does not exist in S/F" or "R is not significant for (essential to) F". This is the primary reason for which the two theories are judged to be different. The difference persists even if we put a restriction on R in $S/F(T)$. The fact that $R = 0$ does not eliminate the ontological difference. This in fact, is the difference between idealization and abstraction [Zielińska, 1981].

Yet it should be remembered that scientists are rarely interested in ontological issues *per se*, thus we may neglect the question of ontic commitment, showing that t can be derived from T when the newly introduced factor, R, is idealized. We thus take the following pragmatic (and metaphysically false) assumption:

(A) To abstract from A is the same as to idealize A.

The space of factors essential for F in t becomes a special case of the space of factors essential for F in T (under (A)), *viz.* when $R = 0$:

$$S/F(T/o) = \langle R = 0, P, Q\rangle = S/F(t) = \langle P, Q\rangle$$

Accordingly, we can derive the laws of t:

[$T/o1$] if $U(T/o)x$ & $R(x) = 0$ & $P(x) = 0$ & $Q(x) = 0$, then $F(x) = 0$[3]

[$T/o2$] if $U(T/o)x$ & $R(x) = 0$ & $P(x) \neq 0$ & $Q(x) = 0$, then $F(x) =$
$= a(k(P(x))$
[$T/o3$] if $U(T/o)x$ & $R(x) = 0$ & $P(x) \neq 0$ & $Q(x) \neq 0$, then $F(x) =$
$= b(k'(P(x), Q(x))$

T/o is indeed very similar to the reduced theory *t*. The only difference is due to the coefficients of *T* (*a*, *b*). This explains the convergence of the empirical results obtained using the reduced and the reducing theories under favourable conditions. When objects fall freely near the surface of the Earth both Galilean and Newtonian theories make good predictions. When the velocities are substantially smaller than the speed of light, classical and relativistic physics make good predictions. It is only in the other values that the factors discovered by the reducing theory adopt, that the explanatory and predictive superiority of the latter theory manifests itself.

It will be thus noticed that the content of the two theories is different, contrary to the positivistic view. It is not possible to derive the reduced theory from the reducing theory unless (A) is assumed. What (A) in fact does is to annihilate the distinctness of the theory.

EXAMPLE 1. Let us consider Galileo's law of free falling bodies. $S/F =$
$= \langle m, R \rangle$, where *m* is mass, *R* is air resistance.

[G_1] if $m \neq 0$ & $R = 0$, then $F = gm$
[G_2] if $m \neq 0$ & $R = 0$, then $F = gm - R$

where *g* is a constant — the gravitational constant. Newton's law of universal gravitation eliminates *g*. It admits two new factor to *S/F' viz.* the mass of the Earth (*M*), and the distance from the centre of the Earth to the centre of the falling object (*r*): $S/F' = \langle M, m, r, R \rangle$.

[N_1] if $M/r^2 \neq 0$ & $m = 0$ & $R = 0$, then $F \approx GM/r^2$
[N_2] if $M/r^2 \neq 0$ & $m \neq 0$ & $R = 0$, then $F = mGM/r^2$
[N_3] if $M/r^2 \neq 0$ & $m \neq 0$ & $R \neq 0$, then $F = mGM/r^2 - R$

To derive [G_1]-[G_2] from [N_1]-[N_2] M/r^2 must adopt a certain value, *e*, which is the ratio of the mass of the Earth to the square of the radius of the Earth at the surface of the Earth.

$[N/e1]$ if $M/r^2 = e$ & $m = 0$ & $R = 0$ then $F \approx g$
$[N/e2]$ if $M/r^2 = e$ & $m \neq 0$ & $R \neq 0$ then $F = gm$
$[N/e3]$ if $M/r^2 = e$ & $m \neq 0$ & $R \neq 0$ then $F = gm - R$

The consequents of the two last statements are identical with the consequents of $[G_1]$-$[G_2]$. It is in this sense that they have been derived from $[N_1]$-$[N_3]$ and the special condition imposed ($M/r^2 = e$)

EXAMPLE 2 (special relativity force and Newtonian force). Consider the classical notion first: $S/F = \langle m, v, t, f \rangle$, where m is mass of the body, v is the velocity of the body, t is the time, f are the forces of friction.

$[n_1]$ if $dv/dt \neq 0$ & $m = 0$ & $f = 0$, then $F \approx dv/dt$
$[n_2]$ if $dv/dt \neq 0$ & $m \neq 0$ & $f = 0$, then $F = d(mv)/dt$
$[n_3]$ if $dv/dt \neq 0$ & $m \neq 0$ & $f \neq 0$, then $F = d(mv)/dt - f$

In Einstein's theory of relativity, on the other hand, we have $S/F = \langle c, \mathrm{v}, v, m, t, f \rangle$, where c is the speed of light, v is the velocity of the observer, v is the velocity of the body relative to the observer.

$[e_1]$ if $\mathrm{v}^2/c^2 \neq 0$ & $dv/dt = 0$ & $m = 0$ & $f = 0$, then $F \approx (1 - \mathrm{v}^2/c^2)$
$[e_2]$ if $\mathrm{v}^2/c^2 \neq 0$ & $dv/dt \neq 0$ & $m = 0$ & $f = 0$, then $F \approx$
$\approx d/dt[v/(1 - \mathrm{v}^2/c^2)]$
$[e_3]$ if $\mathrm{v}^2/c^2 \neq 0$ & $dv/dt \neq 0$ & $m \neq 0$ & $f = 0$, then $F =$
$= d/dt[mv/(1 - \mathrm{v}^2/c^2)]$
$[e_4]$ if $\mathrm{v}^2/c^2 \neq 0$ & $dv/dt \neq 0$ & $m \neq 0$ & $f \neq 0$, then $F =$
$= d/dt[mv/(1 - \mathrm{v}^2/c^2)] - f.$

To derive $[n_1]$-$[n_3]$ we would have to neglect as before the most essential factor for $[e_1]$-$[e_4]$, i.e. assume that $\mathrm{v}^2/c^2 = 0$. Accordingly, the special case of Einstein's theory would be as follows:

$[e_1]$ if $\mathrm{v}^2/c^2 = 0$ & $dv/dt = 0$ & $m = 0$ & $f = 0$, then $F \approx 1$
$[e_2]$ if $\mathrm{v}^2/c^2 = 0$ & $dv/dt \neq 0$ & $m = 0$ & $f = 0$, then $F \approx dv/dt$
$[e_3]$ if $\mathrm{v}^2/c^2 = 0$ & $dv/dt \neq 0$ & $m \neq 0$ & $f = 0$, then $F = d(mv)/dt$
$[e_4]$ if $\mathrm{v}^2/c^2 = 0$ & $dv/dt \neq 0$ & $m \neq 0$ & $f \neq 0$, then $F = d(mv)/dt - f$

As before the consequents of $[e_2]$-$[e_4]$ are identical to the consequents of $[n_1]$-$[n_3]$ respectively.

REDUCTION AND EXPLANATION. The sense in which the reducing theory provides an explanation for the reduced theory is also well seen in terms of idealizational conception of science. For to explain a phenomenon means to show the essence of the phenomenon and show how it is modified (distorted) by the secondary factors, thus obtaining the phenomenon. The reducing theory postulates a wholly new principal factor — thus points to a new essence in a sense. Furthermore, it explains the supposed essence (as conceived by the reduced theory) in terms of the essence it postulates. For it shows how its secondary factors (i.e. the principal and secondary factors of the reduced theory) modify the first idealizational law of the reducing theory.

KINDS OF REDUCTION. Let us briefly consider kinds of reduction possible according to the considerations. The condition that will allow some diversification will be (III). Three independent criteria can distinguish between kinds of reduction:

a) Is the order of factors common to the spaces of factors essential to the investigated variable the same (common sequence) or not (common set)?

b) Is the whole space of factors essential to F in t a part (subset/subsequence) of the space of factors essential to F in T, or are there some factors considered by t which are not considered by T?

c) Are the corrective coefficients of the factors common to both theories the same in both?

The first criterion, (a), yields simple vs. complex reductions. Simple reductions are ones which preserve the essentialist ordering of the factors common to the reduced and reducing theories. Instead of a talking of a conservative space we may talk about a conservative sequence. Complex reductions do not preserve the essentialist ordering of the factors in the conservative space. This may be because the reducing theory not only introduces new principal factors, but also some new secondary factors (this would be the case of joint reduction and correspondence), another possibility would be a change the essentialist ordering of the conservative space. Or both could be the case.

The second criterion, (b) yields subsumptive vs. non-subsumptive reductions. The reduction is subsumptive when the space of factors of the reduced theory is identical with the conservative space. That is to say, the reducing theory includes all the factors that the reduced theory does and some more (the principal factors in case of simple reductions, and principal and some secondary factors in case of the complex reductions). Both Galileo-Newton and Newton-Einstein reductions are cases of subsumptive reductions. The reduction is non-subsumptive when the conservative set is only a subset of the space of factors of the reduced theory, which the reducing theory "does not reduce", so to speak. This case could include interdisciplinary reductions. Whereas the fundamental laws are explained in terms of the reducing theory (science) the specific laws derived from the fundamental law are not.

Finally, (c) yields correctional vs. non-correctional reductions. Non-correctional reductions are reductions where the correctional coefficients of the factors of the conservative space are the same for both theories. Correctional reductions, on the other hand, involve changes in the correctional coefficients of the mentioned factors.

3. *Dialectical correspondence and dialectical reduction*

There is a striking similarity between the dialectical correspondence and reduction. They both consist in "adding" new factors to the space of factors of the old theory, thereby resulting in a new theory which is both different from the old one, and yet similar (the conservative space) to the extent that allows for a continuation, so that some of the explanations are retained. The crucial difference between them is that whereas dialectical correspondence deals with the revision of *secondary* factors, dialectical reduction deals with the revision of the *principal* factors. If dialectical correspondence were to serve as a principle of progress for scientific theories, it was for the gradual progress characteristic of the periods of "normal science", to use Kuhn's term. Dialectical reduction, in this sense is to serve as a principle used in "revolutionary science", whereby one theory is in a sense abolished (the first idealizational laws of such theories differ most acutely), yet there is a continuation of these for the first idealizational law of the reduced theory appears as a close concretization of the first idealizational law of the reducing theory.

The content of two reduced or corresponding theories is different (contrary to the reduction myth) – to say that *T'* dialectically corresponds to *T* is to say that new secondary factors are discovered, to say that *t* is reduced to *T* is to say that new principal factors are discovered. Just as dialectical correspondence also dialectical reduction allows for an incompatibility between theories.

Dialectical correspondence and reduction are thus two distinct concepts — in this sense Nowakowa's thesis is supported. At the same time they constitute two ends of the same spectrum — of dialectical development of theories through the adding of factors, either principal (reduction) or secondary (correspondence). Whereas the former constitutes the principle of development of "normal science", explaining the gradual changes between theories, the latter constitutes the principle of development of "revolutionary science", accounting for the more fundamental changes.

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NOTES

[1]For the exposition of the main notions of the idealizational conception of science, cf. [Nowak, 1980].

[2]In that case we would have a case of correspondence between theories, cf. [Nowakowa, 1974].

[3]It should be noticed that a corresponding statement is not defined in *t*. The only reason why it is defined here is the assumption (A).

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