Workbook Unit 17:

Negated Categorical Propositions

Overview

In this unit, we will continue to learn the basics of symbolization in predicate logic. We will discuss the so-called Negated Categorical Equivalences.

This unit

- teaches you the Negated Categorical Equivalences

Prerequisites

You must have completed Units 13-16.

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1. Reminder

We should be able to symbolize all four categorical propositions

A:	**All** men are jealous	**All** Ss are P
I:	**Some** women are ambitious	**Some** Ss are P
E:	**No** women are obnoxious	**No** Ss are P
O:	**Some** men are **not** crazy	**Some** Ss are **not** P

and you should know the Negated Quantifier Equivalences:

$\sim\forall x\, Px :: \exists x \sim Px$
$\sim\exists x\, Px :: \forall x \sim Px$

2. Negated Categorical Propositions

We will be using the following symbolization key:

U.D.: people
Ax: *x* is ambitious
Cx: *x* is crazy.
Jx: *x* is jealous
Mx: *x* is a man
Ox: *x* is obnoxious
Wx: *x* is a woman

2.1. Negation of Proposition A: Not all Ss are P

Let us first consider an A type proposition "All men are jealous". Let us negate it:

(~A) It is not the case that: all men are jealous.

or more idiomatically:

(~A) Not all men are jealous.
[~A] $\sim\forall x\,(Mx \rightarrow Jx)$

To think that not all men are jealous is surely to think that there is some man who is not jealous. Since the reverse is also true (if someone holds that there is some man who is not jealous, that person also must think that not all men are jealous), the negation of proposition A is equivalent to proposition O:

(O) There is at least one man who is not jealous.
[O] $\exists x\,(Mx \bullet \sim Jx)$

Exercise "Negated Proposition A"

Symbolize the following propositions in two equivalent ways (as negations of a universal proposition and as an existential proposition). State the existential proposition in English.

U.D.: animals

Bx: *x* barks	*Hx*: *x* howls
Cx: *x* is a cat	*Lx*: *x* likes to walk
Dx: *x* is a dog	*Mx*: *x* meows
Fx: *x* likes canned food	*Wx*: *x* wags its tail

		$\sim\forall x\,(Sx \rightarrow Px)$	$\exists x\,(Sx \bullet \sim Px)$
(a)	Not all dogs howl.		
(b)	Not every cat meows.		
(c)	Not all dogs wag their tails.		
(d)	Not all cats like canned food.		
(e)	Not every dog likes to walk.		
(f)	Not all dogs bark.		

2.2. Negation of Proposition E: It is not the case that no Ss are P

Consider an E type proposition "No women are obnoxious". Imagine someone (a man probably), who believes that this proposition is false, in other words, he thinks that:

(~E) It is not the case that: no women are obnoxious.
[~E] $\sim\forall x\ (Wx \rightarrow \sim Ox)$

To think that the claim that no women are obnoxious is false is surely to think that there is some woman who *is* obnoxious. Since the reverse is also true (if someone thinks that there is some woman who is obnoxious, he must think that the claim that no women are obnoxious is false), the negation of proposition E is equivalent to proposition I:

(I) There is at least one woman who is obnoxious.
[I] $\exists x\ (Wx \bullet Ox)$

Exercise "Negated Proposition E"

Symbolize the following propositions in two equivalent ways (as negations of a universal proposition and as an existential proposition). State the existential proposition in English.

U.D.: animals

Ax: *x* likes to stay alone	*Fx*: *x* likes canned food
Cx: *x* is a cat	*Sx*: *x* likes to swim
Dx: *x* is a dog	*Wx*: *x* likes to walk

		$\sim\forall x\ (Sx \rightarrow \sim Px)$	$\exists x\ (Sx \bullet Px)$
(a)	It isn't the case that no cat likes to walk.		
(b)	It is false that no dogs like canned food.		
(c)	It is false that no dog likes to stay alone.		
(d)	It is false that no cats like canned food.		
(e)	The claim that no cat likes to swim is false		
(f)	It is false that no cat likes to stay alone.		

2.3. Negation of Proposition I: There are no Ss that are P

Let us turn to the existential propositions. Let us begin with an I type proposition "There are women who are ambitious". Imagine someone (again, probably, a man), who believes that this proposition is false, in other words, he thinks that:

(~I) It is not the case that: there are women who are ambitious.
[~I] $\sim\exists x\,(Wx \bullet Ax)$

To think that the claim that there are women who are ambitious is false is surely to think that *no woman is ambitious*. Since the reverse is also true (if someone thinks that no women are ambitious, he must think that the claim that some women are ambitious is false), the negation of proposition I is equivalent to proposition E:

[E] $\forall x\,(Wx \rightarrow \sim Ax)$
(E) No woman is ambitious.

Note that the equivalence between (~I) and (E) is so clear that it has even been "encoded" in the language – both propositions can be read "No Ss are P".

Exercise "Negated Proposition I"

Symbolize the following propositions in two equivalent ways (as negations of an existential proposition and as a universal proposition). State the negation of the existential proposition in English.

U.D.: animals

Ax: *x* likes to stay alone	*Fx*: *x* likes canned food
Cx: *x* is a cat	*Sx*: *x* likes to swim
Dx: *x* is a dog	*Wx*: *x* likes to walk

		$\sim\exists x\,(Sx \bullet Px)$	$\forall x\,(Sx \rightarrow \sim Px)$
(a)	No cats like to swim.		
	It is not the case that:		
(b)	No dogs like canned food.		
	It is not the case that:		
(c)	No dog likes to stay alone.		
	It is not the case that:		
(d)	No cats like canned food.		
	It is not the case that:		
(e)	No cat likes to walk.		
	It is not the case that:		
(f)	No cat likes to stay alone.		
	It is not the case that:		

2.4. Negation of Proposition O: There are no Ss that are not P

Finally, consider the O type proposition "There are men who are not crazy". Imagine someone (this time, probably, a woman), who believes that this proposition is false, in other words, she thinks that:

(~O) It is not the case that: there are men who are not crazy.
[~O] $\sim\exists x\ (Mx \bullet \sim Cx)$

To think that the claim that there are men who are not crazy is false is surely to think that *all men are crazy*. Since the reverse is also true (if someone thinks that all men are crazy, she must think that the claim that some men are not crazy is false), the negation of proposition O is equivalent to proposition A:

[A] $\forall x\ (Mx \rightarrow Cx)$
(A) All men are crazy.

Exercise "Negated Proposition O"

Symbolize the following propositions in two equivalent ways (as negations of an existential proposition and as a universal proposition). State the universal proposition in English.

U.D.: animals

Bx: *x* barks	*Hx*: *x* howls
Cx: *x* is a cat	*Lx*: *x* likes to walk
Dx: *x* is a dog	*Mx*: *x* meows
Fx: *x* likes canned food	*Wx*: *x* wags its tail

		$\sim\exists x\ (Sx \bullet \sim Px)$	$\forall x\ (Sx \rightarrow Px)$
(a)	There are no cats that do not meow.		
(b)	There are no dogs that do not bark.		
(c)	There are no dogs that do not howl.		
(d)	There are no dogs that don't wag their tails.		
(e)	There are no dogs that do not like to walk.		
(f)	There are no cats that dislike canned food.		

2.5. Negated Categorical Equivalences

We can thus gather together all the equivalences in a table:

~A	$\sim\forall x\,(Sx \rightarrow Px)$	::	$\exists x\,(Sx \bullet \sim Px)$	O
~E	$\sim\forall x\,(Sx \rightarrow \sim Px)$	::	$\exists x\,(Sx \bullet Px)$	I
~I	$\sim\exists x\,(Sx \bullet Px)$	::	$\forall x\,(Sx \rightarrow \sim Px)$	E
~O	$\sim\exists x\,(Sx \bullet \sim Px)$	::	$\forall x\,(Sx \rightarrow Px)$	A

There is an easy mnemonic that can help you remember that negations of propositions A, E, I, O are equivalent to that same series but in reverse order: O, I, E, A.

Exercise "Negated Categorical Propositions"

Symbolize the following opinions about politicians using the symbolization key provided. In each case, provide two equivalent symbolizations.

U.D.: politicians

Ax: *x* is ambitious
Cx: *x* is corrupt
Dx: *x* is diligent
Hx: *x* is honest
Ix: *x* is intelligent
Nx: *x* is new to politics
Px: *x* is pretentious
Tx: *x* is tired

(a)	No intelligent politicians are corrupt.		
(b)	There are no intelligent politicians who are not honest.		
(c)	Not all corrupt politicians are intelligent.		
(d)	Not all diligent politicians are tired.		
(e)	There are no ambitious politicians who are not honest.		
(f)	No honest politician is corrupt.		
(g)	It is not the case that no diligent politician is corrupt.		
(h)	No corrupt politician is honest.		
(i)	There are no pretentious politicians who are new to politics.		
(j)	Not all honest politicians are tired.		
(k)	It is not the case that no corrupt politician is tired.		
(l)	No honest politician is pretentious.		
(m)	Not only ambitious politicians are corrupt.		

What You Need to Know and Do

- You need to be able to symbolize simple and externally complex singular and quantified propositions.
- You need to be able to construct an appropriate symbolization key (with U.D. given).