Solutions to Workbook Exercises

Unit 16:

Categorical Propositions

Exercise "Categorical Propositions - 1"

(a)	All politicians are nasty.	Proposition:	A
(b)	Some Democrats are not happy after the elections.	Proposition:	O
(c)	Some Republicans are happy after the elections.	Proposition:	I
(d)	No California Democrats are happy.	Proposition:	E
(e)	Some voters are not happy.	Proposition:	O
(f)	Some voters are happy.	Proposition:	I
(g)	All Americans have voted in the elections.	Proposition:	A
(h)	No Americans are pretentious.	Proposition:	E

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Exercise "Categorical Propositions - 2"

Symbolize the following opinions about politicians using the symbolization key provided. For each of the propositions, write down the canonical reading:

U.D.: politicians	*Ax*: *x* is ambitious	*Ix*: *x* is intelligent
	Cx: *x* is corrupt	*Nx*: *x* is new to politics
	Dx: *x* is diligent	*Px*: *x* is pretentious
	Hx: *x* is honest	*Tx*: *x* is tired

(a)	Some intelligent politicians are corrupt.	$\exists x\,(Ix \bullet Cx)$
	There is an *x* such that *x* is intelligent and *x* is corrupt.	
(b)	There is an intelligent politician who is honest.	$\exists x\,(Ix \bullet Hx)$
	There is an *x* such that *x* is intelligent and *x* is honest.	
(c)	Some corrupt politicians are intelligent.	$\exists x\,(Cx \bullet Ix)$
	There is an *x* such that *x* is corrupt and *x* is intelligent.	
(d)	Some corrupt politicians are not intelligent.	$\exists x\,(Cx \bullet \sim Ix)$
	There is an *x* such that *x* is corrupt and *x* is not intelligent.	
(e)	Some ambitious politicians are not honest.	$\exists x\,(Ax \bullet \sim Hx)$
	There is an *x* such that *x* is ambitious and *x* is not honest.	
(f)	All corrupt politicians are ambitious.	$\forall x\,(Cx \rightarrow Ax)$
	For every *x*, if *x* is corrupt then *x* is ambitious.	
(g)	Any politician who is new to politics is honest.	$\forall x\,(Nx \rightarrow Hx)$
	For every *x*, if *x* is new to politics then *x* is honest.	
(h)	No corrupt politicians are honest.	$\forall x\,(Cx \rightarrow \sim Hx)$
	For every *x*, if *x* is corrupt then *x* is not honest.	
(i)	No honest politician is corrupt.	$\forall x\,(Hx \rightarrow \sim Cx)$
	For every *x*, if *x* is honest then *x* is not corrupt.	
(j)	All honest politicians are tired.	$\forall x\,(Hx \rightarrow Tx)$
	For every *x*, if *x* is honest then *x* is tired.	
(k)	No politician who is new to politics is tired.	$\forall x\,(Nx \rightarrow \sim Tx)$
	For every *x*, if *x* is new to politics then *x* is not tired.	
(l)	No honest politician is pretentious.	$\forall x\,(Hx \rightarrow \sim Px)$
	For every *x*, if *x* is honest then *x* is not pretentious.	

Exercise "Categorical Propositions - 3"

Symbolize the following propositions. For each of the propositions, write down the canonical interpretation.

U.D.: animals

Bx: *x* barks	*Hx*: *x* howls
Cx: *x* is a cat	*Lx*: *x* likes to walk
Dx: *x* is a dog	*Mx*: *x* meows
Fx: *x* likes canned food	*Wx*: *x* wags its tail

(a)	Some dogs howl.	$\exists x\,(Dx \bullet Hx)$
	There is an *x* such that *x* is a dog and *x* howls.	
(b)	No cats howl.	$\forall x\,(Cx \rightarrow \sim Hx)$
	For every *x*, if *x* is a cat then *x* does not howl.	
(c)	Some animals howl.	$\exists x\,Hx$
	There is an *x* such that *x* howls.	
(d)	Some cats do not like canned food.	$\exists x\,(Cx \bullet \sim Fx)$
	There is an *x* such that *x* is a cat and *x* does not like canned food.	
(e)	All cats meow.	$\forall x\,(Cx \rightarrow Mx)$
	For every *x*, if *x* is a cat then *x* meows.	
(f)	No cat likes to walk.	$\forall x\,(Cx \rightarrow \sim Lx)$
	For every *x*, if *x* is a cat then *x* does not like to walk.	
(g)	All dogs wag their tails.	$\forall x\,(Dx \rightarrow Wx)$
	For every *x*, if *x* is a dog then *x* wags its tail.	
(h)	All animals like to walk.	$\forall x\,Lx$
	For every *x*, *x* likes to walk.	

Exercise “Free and Bound Variables”

Show which variables are free and determine whether the formula is a proposition or a propositional function.

(a) $\forall x\, Px$

☑ proposition
❑ propositional function

(b) $\forall x\, (Px \bullet Qx)$

☑ proposition
❑ propositional function

(c) $\forall x\, (Px \bullet Qx) \rightarrow Rx$

❑ proposition
☑ propositional function

(d) $\forall x\, Px \bullet Qx$

❑ proposition
☑ propositional function

(e) $\exists x\, Px \equiv Qx$

❑ proposition
☑ propositional function

(f) $\exists x\, {\sim}Px \bullet Qx$

❑ proposition
☑ propositional function

(g) $\exists x\, ({\sim}Px \bullet Qx)$

☑ proposition
❑ propositional function

(h) $\forall x\, (Px \bullet Qx) \rightarrow {\sim}(Px \bullet Rx)$

❑ proposition
☑ propositional function

(i) $\exists x\, {\sim}(Px \bullet Qx)$

☑ proposition
❑ propositional function

(j) $\exists x\, ({\sim}(Px \rightarrow Qx) \bullet {\sim}(Px \bullet Rx))$

☑ proposition
❑ propositional function

(k) $\exists x\, ({\sim}(Px \rightarrow Qx) \bullet {\sim}(Px \bullet Rx)) \vee {\sim}(Rx \rightarrow Cx)$

❑ proposition
☑ propositional function

Exercise " 'Only' Propositions – 1"

Symbolize the following propositions using the symbolization key provided. For each of the propositions, write down the canonical interpretation.

U.D.: animals	*Bx*: *x* barks	*Hx*: *x* howls
	Cx: *x* is a cat	*Lx*: *x* likes to walk
	Dx: *x* is a dog	*Mx*: *x* meows
	Fx: *x* likes canned food	*Wx*: *x* wags its tail

(a) Only dogs bark. $\forall x\,(Bx \rightarrow Dx)$

For every *x*, if *x* barks then *x* is a dog.

(b) Only cats meow. $\forall x\,(Mx \rightarrow Cx)$

For every *x*, if *x* meows then *x* is a cat.

(c) Only dogs howl. $\forall x\,(Hx \rightarrow Dx)$

For every *x*, if *x* howls then *x* is a dog.

(d) Only dogs wag their tails $\forall x\,(Wx \rightarrow Dx)$

For every *x*, if *x* wags its tail then *x* is a dog.

(e) Only dogs like to walk. $\forall x\,(Lx \rightarrow Dx)$

For every *x*, if *x* likes to walk then *x* is a dog.

(f) Only cats like canned food. $\forall x\,(Fx \rightarrow Cx)$

For every *x*, if *x* likes canned food then *x* is a cat.

(g) Only animals that bark like to walk. $\forall x\,(Lx \rightarrow Bx)$

For every *x*, if *x* likes to walk then *x* barks.

(h) Only animals that like to walk wag their tails. $\forall x\,(Wx \rightarrow Lx)$

For every *x*, if *x* wags its tail then *x* likes to walk.

(i) Only cats do not like to walk. $\forall x\,(\sim Lx \rightarrow Cx)$

For every *x*, if *x* does not like to walk then *x* is a cat.

(j) Only animals that meow like canned food. $\forall x\,(Fx \rightarrow Mx)$

For every *x*, if *x* likes canned food then *x* meows.

Exercise " 'Only' Propositions – 2"

U.D.: people

Fx: *x* is a father
Mx: *x* is a man
Ox: *x* is a mother
Sx: *x* wears skirts
Tx: *x* wears ties.
Wx: *x* is a woman

(a)	All men are fathers.	$\forall x\,(Mx \rightarrow Fx)$	❑ true ☑ false
(b)	Only men are fathers.	$\forall x\,(Fx \rightarrow Mx)$	☑ true ❑ false
(c)	All women are mothers.	$\forall x\,(Wx \rightarrow Ox)$	❑ true ☑ false
(d)	Only women are mothers	$\forall x\,(Ox \rightarrow Wx)$	☑ true ❑ false
(e)	All mothers are women.	$\forall x\,(Ox \rightarrow Wx)$	☑ true ❑ false
(f)	Only mothers are women.	$\forall x\,(Wx \rightarrow Ox)$	❑ true ☑ false
(g)	Only men wear ties.	$\forall x\,(Tx \rightarrow Mx)$	❑ true ☑ false
(h)	Only women wear skirts.	$\forall x\,(Sx \rightarrow Wx)$	☑ true ❑ false
(i)	Only women do not wear ties.	$\forall x\,(\sim Tx \rightarrow Wx)$	❑ true ☑ false
(j)	Only men do not wear skirts.	$\forall x\,(\sim Sx \rightarrow Mx)$	❑ true ☑ false
(k)	Only persons wearing skirts are women	$\forall x\,(Wx \rightarrow Sx)$	❑ true ☑ false
(l)	Only persons wearing ties are men.	$\forall x\,(Mx \rightarrow Tx)$	❑ true ☑ false
(m)	Only persons wearing skirts do not wear ties.	$\forall x\,(\sim Tx \rightarrow Sx)$	❑ true ☑ false
(n)	Only persons who are not mothers are fathers.	$\forall x\,(Fx \rightarrow \sim Mx)$	☑ true ❑ false
(o)	Only persons who do not wear skirts are fathers.	$\forall x\,(Fx \rightarrow \sim Sx)$	☑ true ❑ false