Workbook Unit 14:

Singular and Quantified Propositions

Overview

In this unit, we will introduce a new logical theory called predicate logic. Predicate logic is an extension of propositional logic – we will preserve the logical connectives but we will be delving deeper into the structure of propositions. In this unit, we will introduce two very basic types of propositions there are in predicate logic, viz. singular and quantified propositions.

This unit

- teaches you in what sense predicate logic is an extension of propositional logic
- teaches you to symbolize singular and basic quantified propositions

Prerequisites

You should review your grasp of the logical connectives (Units 2 and 3). Look at the Propositional Logic Symbolization Review Sheet.

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Propositional Logic Symbolization Review Sheet

~	•	∨	→	≡
It's not the case that... Not ... It is false that... It is a lie that ... ...failed...	... and ... both ... and ... ... as well as ... ... but ... ... however ... ... though ... ... although ... ... even though ... ... nevertheless ... ... still ... ... but still ... ... also ... ... while ... ... despite the fact that ... ... moreover ... ... in addition ... ...; ... ..., ...	... or ... either ... or ... ... or else ...	... if ... ...provided that ... ... given that ... ... in case ... ... in the event that ... ... as long as ... ...assuming that ... ...supposing that ... ...on the condition that ... ...on the assumption that ...	... if and only if ... ... if but only if... ... when and only when ... ... just in case ... ... iff ... ... exactly if ...

Neither *p* nor *r*	$\sim p \bullet \sim r$	$\sim(p \vee r)$
Not both *p* and *r*	$\sim(p \bullet r)$	$\sim p \vee \sim r$
Both not *p* and not *r*	$\sim p \bullet \sim r$	$\sim(p \vee r)$
Either *p* or *r* but not both	$(p \vee r) \bullet \sim(p \bullet r)$	
r unless *p*	$p \vee r$	$\sim p \rightarrow r$

p if *r*	if *r* then *p*	*r* is a sufficient condition for *p*	$r \rightarrow p$
p only if *r*	if *p* then *r* [if not *r* then not *p*]	*r* is a necessary condition for *p*	$p \rightarrow r$ $[\sim r \rightarrow \sim p]$
p if and only if *r*		*r* is a necessary and a sufficient condition for *p*	$r \equiv p$

1. Limitations of Propositional Logic

Both of the arguments below are clearly valid:

(A_1) If Socrates is human then he is mortal.
Socrates is human.
———————————
So: Socrates is mortal.

(A_2) All humans are mortal.
Socrates is human.
———————————
So: Socrates is mortal.

It turns out, however, that only argument (A_1) is valid in propositional logic. It becomes clearer when we symbolize both arguments in propositional logic.

$$[A_1] \quad \begin{array}{l} H \rightarrow M \\ \underline{H \qquad} \\ M \end{array}$$

H: Socrates is human
M: Socrates is mortal

The symbolization makes the fact that [A_1] is logically valid only clearer – the argument is simply an instance of Modus Ponens.

$$\begin{array}{l} p \rightarrow q \\ \underline{p \qquad} \\ q \end{array}$$

Let's turn to argument (A_2). The second premise and the conclusion of the arguments are the same but what about the first premise "All humans are mortal"? How can we symbolize it? Well, in light of propositional logic, the first premise is simply another proposition that is quite unrelated to the others (this is what is problematic, as we will see in a moment). We simply need another propositional letter and let it stand for the proposition. Argument (A_2) can only be symbolized thus:

$$[A_2] \quad \begin{array}{l} A \\ \underline{H \qquad} \\ M \end{array}$$

A: All humans are mortal
H: Socrates is human
M: Socrates is mortal

It is important to be clear about this. The proposition "All humans are mortal" is a simple proposition in propositional logic because it is not composed of any other propositions by means of any of the five logical connectives. If so, however, then [A_2] does not instantiate a valid reasoning pattern. The logical form of the argument is this:

$$\begin{array}{l} p \\ \underline{q \quad} \\ r \end{array}$$

(You can construct a truth table to show that this argument form is invalid – there is a row of the truth table where both premises are true while the conclusion is false.)

What is the conclusion to be drawn from this?

Well, the conclusion is not to be that argument (A_2) is invalid. The conclusion is that propositional logic is too weak a theory to understand the validity of that argument. This is in part why a stronger theory, predicate logic, has been proposed.

Acquiring a proper understanding of why argument (A_2) is valid in predicate logic will require some time, but for now it might be sufficient just to draw attention to the major point of difference between propositional and predicate logic. Predicate logic differs from propositional logic in that it affords a special place to propositions such as "All humans are mortal", which are called quantified propositions and which are to be distinguished from propositions such as "Socrates is human" or "Socrates is mortal", which are singular propositions. It is this distinction that escapes propositional logic. In other words, predicate logic provides us with finer tools to look at the structure of propositions.

2. Basic Symbolizations in Predicate Logic

Predicate logic equips us with a stronger (but also more complicated) conceptual apparatus. It will take us three units just to learn the basics. Before we start introducing this apparatus it will pay to look at four propositions.

(1) Cliff is tall.
(2) Alice is tall.
(3) Everybody is tall.
(4) Somebody is tall.

From the point of view of our old propositional logic, all these propositions are simple and their symbolization:

$[1]_{\text{prop.l.}}$	C	C: Cliff is tall.
$[2]_{\text{prop.l.}}$	A	A: Alice is tall.
$[3]_{\text{prop.l.}}$	E	E: Everybody is tall.
$[4]_{\text{prop.l.}}$	S	S: Somebody is tall.

does not reveal the intuitively clear structural connections between the propositions. After all we say (or predicate) *the same* about different subjects, where in addition the subjects in (1)-(2) are individual persons while in (3)-(4) we talk more generally about everybody or somebody.

Predicate logic allows us to see these similarities. Let us see this more concretely by analyzing these propositions in two groups: propositions (1)-(2) are simple singular propositions, while (3)-(4) are simple quantified propositions.

2.1. Simple Singular Propositions

Both propositions (1) and (2) are singular propositions since they are proposition about a certain individual, viz. about Cliff and about Alice.

(1) Cliff is tall.
(2) Alice is tall.

Consider proposition (1). The proposition can be broken into two components thus:

Cliff is tall

Cliff $\qquad\qquad$ $\boxed{}$ is tall

The first of these components is a **name** of an individual, in our case a name of a person. The second component is the so-called **propositional function**, which after the empty space (symbolized by the box) has been filled with a name becomes a proposition. Every propositional function has at least one such empty space, which it is usual to mark by means of the so-called individual variables (x, y, z):

x is tall

Let us construct a symbolization legend. In predicate logic, the symbolization legend includes three parts: the universe of discourse, the list of individual names, the list of propositional functions:

- Each symbolization legend will have an entry called "the universe of discourse" or "U.D." for short. We will talk about it more but, for now, let us only say that the universe of discourse is the set it includes all the individuals we would want to talk about (e.g. all Americans, all people, all students, all physical objects, everything). In our case, the U.D. is the set of people.
- We need a list of individual names of objects talked about. In our case, we are saying something about Cliff. Individual names are abbreviated by small letters from the beginning of the alphabet: *a*, *b*, *c*, *d*, *e*, etc. We can choose '*c*' to abbreviate 'Cliff' and '*a*' to abbreviate 'Alice'.
- Finally, we need a list of propositional functions. Here the propositional function used is 'x is tall', which will be abbreviated as 'Tx'. Note that the abbreviations of propositional functions are composed of a capital letter, which should stand as a reminder of the property that is being predicate, i.e. for "is tall" and immediately following it the name of the variable x.

Our symbolization legend will look thus:

U.D.: people
a: Alice
c: Cliff
Tx: *x* is tall

We can then put the proposition (1)

(1) Cliff is tall

in terms of predicate logic as:

[1] *Tc*

You should think of the name as having "jumped" in place of '*x*'.

We can offer an analogical rendition of proposition (2)

(2) Alice is tall.

as

[2] ________

It will get some getting used to on your part to recognize constructs such as *Tc* or *Ta* as propositions. So, you should do the following exercise immediately.

Exercise "Simple Singular Propositions"

Symbolize the following opinions about U.S. politicians using the symbolization key provided:

U.D.: U.S. politicians	*b*: Bill Clinton	*Ax*: *x* is ambitious
	c: Condoleeza Rice	*Ix*: *x* is intelligent
	g: George W. Bush	*Px*: *x* is pretentious
	h: Hillary Clinton	*Wx*: *x* is warm

(a) Bill Clinton is a warm man.

(b) Hillary Clinton is also very warm.

(c) Condoleeza Rice is ambitious.

(d) George W. Bush is also very ambitious.

(e) George W. Bush, however, is primarily intelligent.

(f) Bill Clinton is ambitious.

(g) Hillary Clinton is likewise ambitious.

(h) Bill Clinton is a pretentious person.

(i) Hillary Clinton is the quintessence of pretentiousness.

(j) Condoleeza Rice is intelligent.

(k) Condoleeza Rice is warm.

(l) George W. Bush is warm too.

2.2. Simple Quantified Propositions

In order to symbolize propositions (3) and (4), we need to introduce special operators, the so-called quantifiers.

(3) Everybody is tall.
(4) Somebody is tall.

There are two quantifiers: the universal quantifier and the existential quantifier. The **universal quantifier** corresponds to the English phrases ‘everybody’, ‘everything’, ‘all’, ‘for all’, ‘any’ (in most occurrences), and the like. We will represent it by the inverted letter ‘A’ followed by the name of the variable it binds. The form of a quantified proposition created by means of the universal quantifier is:

$\forall x\, P\, x$ read: “for every x, P of x”

The **existential quantifier** corresponds to the English phrases ‘there is some’, ‘there is at least one’, ‘somebody’, ‘something’, ‘there exists’, and the like. We will represent it by an inverted letter ‘E’ followed by the name of the variable it binds. The form of a quantified proposition created by means of the existential quantifier is:

$\exists x\, P\, x$ read: “there is an x such that P of x” or
“for some x, P of x”.

Using our previous legend:

U.D.: people
Tx: x is tall

we can symbolize propositions (3)-(4) respectively as:

[3] $\forall x\, Tx$ “For every x, x is tall”
[4] $\exists x\, Tx$ “There is an x such that x is tall”

Propositions (3) and (4) are quantified propositions. They are also called “general propositions,” though they are “general” in a technical sense. We usually use the term ‘general’ in such a way that only proposition (3) would be counted as a general proposition.

Exercise "Simple Quantified Propositions - 1"

Symbolize the following propositions. Mark the English expressions that correspond to the quantifiers:

U.D.: people	Bx: x is beautiful	Ix: x is intelligent	Vx: x is evil
	Gx: x is good	Lx: x is a liar	Ux: x is ugly

(a) Everyone is beautiful.

(b) Everyone is intelligent.

(c) Somebody is intelligent.

(d) There are intelligent people.

(e) There is a person who is intelligent.

(f) There is a person who is a liar.

(g) All people are good.

(h) All people are evil.

(i) Someone is ugly.

(j) Everybody is ugly.

(k) There are ugly people.

Exercise "Simple Quantified Propositions - 2"

Symbolize the following opinions about politicians. Note that sometimes it is impossible to fully adequately capture the content of the opinions.

U.D.: politicians	Hx: x is honest	Lx: x is a liar
	Ix: x is intelligent	Rx: x is articulate

(a) All politicians are liars.

(b) Politicians are a bunch of liars.

(c) There are honest politicians.

(d) Some politicians are liars.

(e) Any given politician is a liar.

(f) Politicians are intelligent without exception.

(g) Intelligent politicians do happen.

(h) Every politician is articulate.

(i) Rarely does one meet an honest person among politicians.

(j) At least one politician is intelligent.

3. Internally and Externally Complex Propositions

Predicate logic allows for quite complex constructions. We will be introducing them slowly, but it might be worthwhile for us to have an overview of what is in stalk.

In propositional logic, we distinguished five kinds of complex propositions: negations, conjunctions, disjunctions, conditionals and biconditionals. In predicate logic, the five connectives are also present but they may bind not only (a) propositions (both singular and quantified), but also (b) propositional functions. In virtue of the fact (a) that connectives may bind propositions, propositions can be externally simple and externally complex (or: simple and complex). In virtue of the fact (b) that connectives may also bind propositional functions, propositions may be internally simple or internally complex. The following diagram is a simplified classification of the kinds of propositions there are in predicate logic:

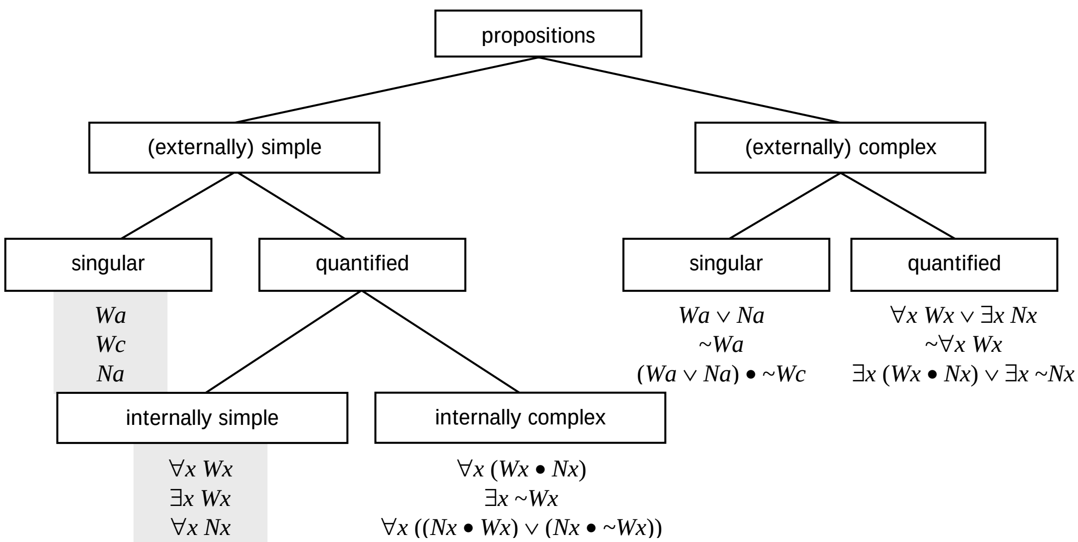

Fig. 1. A simplified classification of propositions in predicate logic (see text)

For now, just glance at the diagram and treat it as a map. So far we have talked about (externally) simple singular propositions and about externally simple and internally simple quantified propositions (highlighted in the diagram). In this unit, we will still look at externally complex singular and quantified propositions.

4. (Externally) Complex Propositions

Externally complex propositions are those that are created by means of the five logical connectives. The logical connectives bind propositions. Since there are two types of propositions, there can be two types of homogeneous complex propositions: singular and quantified.

4.1. (Externally) Complex Singular Propositions

Complex singular propositions are created from other singular propositions by means of propositional connectives. Here are some examples.

[1] $\sim(Va \bullet La)$
[2] $\sim Va \bullet La$

which, given the following symbolization key:

U.D.: people
a: Adam
Lx: *x* will go to hell
Vx: *x* will go to heaven.

symbolize the following propositions:

(1) It is not the case that Adam will go both to heaven and to hell. (or: Adam will not go both to heaven and to hell.)
(2) Adam will not go heaven but to hell.

Exercise "Complex Singular Propositions"

U.D.: U.S. politicians

b: Bill Clinton
c: Condoleeza Rice
g: George W. Bush
h: Hillary Clinton

Hx: *x* is honest
Ix: *x* is intelligent
Lx: *x* is a liar
Rx: *x* is articulate

(a)	Bill Clinton is articulate and intelligent.	
(b)	Bill Clinton is articulate but not intelligent.	
(c)	If Hillary Clinton is intelligent then so is Bill Clinton.	
(d)	Hillary Clinton is a liar just in case she is not honest.	
(e)	Neither George W. Bush nor Condoleeza Rice are liars.	
(f)	George W. Bush and Condoleeza Rice are not both honest.	
(g)	If neither Bill Clinton nor Hillary Clinton are liars and George W. Bush is honest, then Condoleeza Rice is the liar.	
(h)	Hillary Clinton is either honest or an extra-ordinarily articulate and intelligent liar.	
(i)	All four (Bill Clinton, Condoleeza Rice, George W. Bush, Hillary Clinton) are intelligent.	
(j)	At least one of the four is a liar.	

4.2. Externally Complex Quantified Propositions

Propositional connectives can also bind quantified propositions in any combinations. We will be still working with the symbolization key:

U.D.: people
Lx: *x* will go to hell
Vx: *x* will go to heaven.

Consider another example of a complex quantified proposition:

[3] $\forall x\, \boxed{Lx} \rightarrow \exists x\, \boxed{Lx}$

Proposition [3] is a conditional whose antecedent is the universal proposition "Everybody will go to hell" and whose consequent is the existential proposition "Somebody will go to hell". Putting these ingredients together we can read [3] as:

(3) If everybody will go to hell then somebody will go to hell.

Let us now look at the negations of quantified propositions. First, consider the **negation of a universal proposition:**

[4] $\sim\forall x\, \boxed{Vx}$

[4] is a negation of the proposition "Everybody will go to heaven". This means that we can read proposition [4] thus:

It is not the case that: everybody will go to heaven.

or more idiomatically:

(4) Not everybody will go to heaven.

Consider the **negation of an existential proposition**:

[5] $\sim\exists x\, \boxed{Lx}$

[5] is a negation of the proposition "Somebody will go to hell" or "There is a person who will go to hell". This means that we can read proposition [5] thus:

It is not the case that: somebody will go to hell, or:
It is not the case that: there is a person who will go to hell, or:
There is no person who will go to hell,

or more idiomatically:

(5) Nobody will go to hell.

Of course, propositions can be quite complex. Consider the following:

(6) If nobody goes to hell then nobody goes to heaven.

This is a conditional whose antecedent "nobody goes to hell" is a negated existential proposition, and whose consequent "nobody goes to heaven" is likewise a negated existential proposition. We thus have:

[6] $\sim\exists x\, Lx \rightarrow \sim\exists x\, Vx$

Exercise “Externally Complex Quantified Propositions - 1”

What English propositions are symbolized by the following logical formulas?

U.D.: people   *Lx*: *x* goes to hell   *Vx*: *x* goes to heaven

(a) $[\exists x]\, Vx \rightarrow [\forall x]\, Vx$

(b) $[\forall x]\, Vx \lor [\forall x]\, Lx$

(c) $[\exists x]\, Vx \bullet [\exists x]\, Lx$

(d) $[\forall x]\, Lx \equiv [\forall x]\, Vx$

Exercise “Externally Complex Quantified Propositions - 2”

Symbolize the following propositions:

U.D.: people   *Lx*: *x* goes to hell   *Vx*: *x* goes to heaven

(a) Everybody will go to heaven.

(b) Not everybody will go to heaven.

(c) Somebody will go to hell.

(d) Nobody will go to hell.

(e) Not everybody will go to hell.

(f) No person will go to heaven.

(g) If somebody goes to hell then not everybody goes to heaven.

(h) If not everybody goes to hell then somebody goes to heaven .

(i) Somebody goes to heaven and somebody goes to hell.

(j) Nobody goes to heaven and nobody goes to hell.

(j) Either not everybody goes to heaven or not everybody goes to hell.

(k) Nobody goes to heaven just in case nobody goes to hell.

What You Need to Know and Do

- You need to be able to symbolize simple and externally complex singular and quantified propositions.
- You need to be able to construct an appropriate symbolization key (with U.D. given).

Further Reading

You can read about these matters further in a number of logic textbooks. I enclose the chapters titles for the textbooks I have chosen as optional.
Klenk: Ch. 10. Singular Sentences and Ch. 11.1 Universal and Existential Quantifiers
Copi & Cohen: Ch. 10.1. Singular Propositions, Ch.10.2. Quantification.
Hurley: Ch 8.1. Symbols and Translation