

Solutions to Workbook Exercises

Unit 13:

Natural Deduction Proofs (IV)

Examples of proofs:

Example 1

1.	$(A \rightarrow B) \cdot (B \rightarrow A)$	Pr.
2.	A	Assp (\equiv Int)
3.	A \rightarrow B	\bullet Elim 1
4.	B	\rightarrow Elim 3, 2
5.	B	Assp (\equiv Int)
6.	B \rightarrow A	\bullet Elim 1
7.	A	\rightarrow Elim 6, 5
8.	A \equiv B	\equiv Int 2-4, 5-7

Example 2

1.	A \equiv B	Pr.
2.	C \equiv D	Pr.
3.	A \cdot C	Assp (\equiv Int)
4.	A	\bullet Elim 3
5.	B	\equiv Elim 1, 4
6.	C	\bullet Elim 3
7.	D	\equiv Elim 2, 6
8.	B \cdot D	\bullet Int 5, 7
9.	B \cdot D	Assp (\equiv Int)
10.	B	\bullet Elim 9
11.	A	\equiv Elim 1, 10
12.	D	\bullet Elim 9
13.	C	\equiv Elim 2, 12
14.	A \cdot C	\bullet Int 11, 13
15.	(A \cdot C) \equiv (B \cdot D)	\equiv Int 3-8, 9-14

Example 3

1.	A \vee B	Pr.
2.	A \equiv (C \cdot D)	Pr.
3.	B \rightarrow (D \cdot G)	Pr.
4.	A	Assp (\vee Elim)
5.	C \cdot D	\equiv Elim 2, 4
6.	D	\bullet Elim 5
7.	B	Assp (\vee Elim)
8.	D \cdot G	\rightarrow Elim 3, 7
9.	D	\bullet Elim 8
10.	D	\vee Elim 1, 4-6, 7-9

Example 4

1.	(A \cdot B) \rightarrow C	Pr.
2.	B \cdot \sim C	Pr.
3.	A	Assp (\sim Int)
4.	B	\bullet Elim 2
5.	A \cdot B	\bullet Int 3, 4
6.	C	\rightarrow Elim 1, 5
7.	\sim C	\bullet Elim 2
8.	\sim A	\sim Int 3-6, 3-7

Example 5

1.	\sim A \rightarrow B	Pr.
2.	B \rightarrow A	Pr.
3.	\sim A	Assp (\sim Elim)
4.	\sim A	R3
5.	B	\rightarrow Elim 1, 4
6.	A	\rightarrow Elim 2, 5
7.	A	\sim Elim 3-6, 3-4

≡Int.I.a.

(a)

3	B	Assp (≡Int)
5	A	
6	A	Assp (≡Int)
8	B	
9	B ≡ A	≡Int 3-5, 6-8

(b)

3	C	Assp (≡Int)
5	~A	
6	~A	Assp (≡Int)
8	C	
9	C ≡ ~A	≡Int 3-5, 6-8

(c)

3	B → C	Assp (≡Int)
5	C → B	
6	C → B	Assp (≡Int)
8	B → C	
9	(B → C) ≡ (C → B)	≡Int 3-5, 6-8

(d)

3	A ∨ C	Assp (≡Int)
5	~C → A	
6	~C → A	Assp (≡Int)
8	A ∨ C	
9	(A ∨ C) ≡ (~C → A)	≡Int 3-5, 6-8

→Int.I.b.

(a)

3	D	Assp (≡Int)
5	A	
6	A	Assp (≡Int)
8	D	
9	D ≡ A	≡Int 3-5, 6-8

(b)

3	A → C	Assp (≡Int)
5	C → B	
6	C → B	Assp (≡Int)
8	A → C	
9	(A → C) ≡ (C → B)	≡Int 3-5, 6-8

(c)

3	~~B	Assp (≡Int)
5	~(A • ~B)	
6	~(A • ~B)	Assp (≡Int)
8	~~B	
9	~~B ≡ ~(A • ~B)	≡Int 3-5, 6-8

(d)

3	A ≡ B	Assp (≡Int)
5	C	
6	C	Assp (≡Int)
8	A ≡ B	
9	(A ≡ B) ≡ C	≡Int 3-5, 6-8

≡Int.II.

(a) Prove that: $C \equiv D$

1.	$A \vee C$	Pr.
2.	$D \bullet \sim A$	Pr.
3.	C	Assp (\equiv Int)
4.	D	\bullet Elim 2
5.	D	Assp (\equiv Int)
6.	$\sim A$	\bullet Elim 2
7.	C	DS 1, 6
8.	$C \equiv D$	\equiv Int 3–4, 5–7

(b) Prove that: $\sim A \equiv \sim C$

1.	$\sim A \vee C$	Pr.
2.	$A \vee \sim C$	Pr.
3.	$\sim A$	Assp (\equiv Int)
4.	$\sim C$	DS 2, 3
5.	$\sim C$	Assp (\equiv Int)
6.	$\sim A$	DS 1, 5
7.	$\sim A \equiv \sim C$	\equiv Int 3–4, 5–6

(c) Prove that: $A \equiv C$

1.	$A \equiv B$	Pr.
2.	$B \equiv C$	Pr.
3.	A	Assp (\equiv Int)
4.	B	\equiv Elim 1, 3
5.	C	\equiv Elim 2, 4
6.	C	Assp (\equiv Int)
7.	B	\equiv Elim 2, 6
8.	A	\equiv Elim 1, 7
9.	$A \equiv C$	\equiv Int 3–5, 6–8

(d) Prove that: D

1.	$(A \equiv A) \rightarrow B$	Pr.
2.	$B \equiv D$	Pr.
3.	A	Assp (\equiv Int)
4.	A	R 3
5.	$A \equiv A$	\equiv Int 3–4, 3–4
6.	B	\rightarrow Elim 1, 5
7.	D	\equiv Elim 2, 6

\vee Elim.I.a.

(a)

2	$B \vee A$	
3	B	Assp (\vee Elim)
5	C	
6	A	Assp (\vee Elim)
8	C	
9	C	\vee Elim 2, 3–5, 6–8

(b)

2	$\sim C \vee D$	
3	$\sim C$	Assp (\vee Elim)
5	A	
6	D	Assp (\vee Elim)
8	A	
9	A	\vee Elim 2, 3–5, 6–8

(c)

2	$(D \rightarrow A) \vee (A \vee C)$	
3	$D \rightarrow A$	Assp (\vee Elim)
5	$\sim B \equiv A$	
6	$A \vee C$	Assp (\vee Elim)
8	$\sim B \equiv A$	
9	$\sim B \equiv A$	\vee Elim 2, 3–5, 6–8

(d)

2	$\sim A \vee \sim B$	
3	$\sim A$	Assp (\vee Elim)
5	$B \rightarrow A$	
6	$\sim B$	Assp (\vee Elim)
8	$B \rightarrow A$	
9	$B \rightarrow A$	\vee Elim 2, 3–5, 6–8

\vee Elim.I.b.

(a)

2	$D \vee B$	
3	D	Assp (\vee Elim)
5	C	
6	B	Assp (\vee Elim)
8	C	
9	C	\vee Elim 2, 3–5, 6–8

(b)

2	$A \vee B$	
3	A	Assp (\vee Elim)
5	$C \vee A$	
6	B	Assp (\vee Elim)
8	$C \vee A$	
9	$C \vee A$	\vee Elim 2, 3–5, 6–8

(c)

2	$C \vee (A \vee B)$	
3	C	Assp (\vee Elim)
5	$\sim D \vee \sim B$	
6	$A \vee B$	Assp (\vee Elim)
8	$\sim D \vee \sim B$	
9	$\sim D \vee \sim B$	\vee Elim 2, 3–5, 6–8

(d)

2	$(A \rightarrow B) \vee (B \rightarrow A)$	
3	$A \rightarrow B$	Assp (\vee Elim)
5	$A \vee B$	
6	$B \rightarrow A$	Assp (\vee Elim)
8	$A \vee B$	
9	$A \vee B$	\vee Elim 2, 3–5, 6–8

\vee Elim.I.c.

(a)

2	$A \vee \sim A$	
3	A	Assp (\vee Elim)
5	C	
6	$\sim A$	Assp (\vee Elim)
8	C	
9	C	\vee Elim 2, 3–5, 6–8

(b)

2	$\sim B \vee \sim \sim D$	
3	$\sim B$	Assp (\vee Elim)
5	$\sim \sim C$	
6	$\sim \sim D$	Assp (\vee Elim)
8	$\sim \sim C$	
9	$\sim \sim C$	\vee Elim 2, 3–5, 6–8

(c)

2	$(A \vee B) \vee (B \equiv C)$	
3	$A \vee B$	Assp (\vee Elim)
5	$\sim B$	
6	$B \equiv C$	Assp (\vee Elim)
8	$\sim B$	
9	$\sim B$	\vee Elim 2, 3–5, 6–8

(d)

2	$(A \bullet B) \vee (B \bullet A)$	
3	$A \bullet B$	Assp (\vee Elim)
5	$B \bullet A$	
6	$B \bullet A$	Assp (\vee Elim)
8	$B \bullet A$	
9	$B \bullet A$	\vee Elim 2, 3–5, 6–8

\vee Elim.II.

(a) Prove that: $D \vee G$

1.	$\sim A \vee B$	Pr.
2.	$\sim A \rightarrow B$	Pr.
3.	$\sim A$	Assp (\vee Elim)
4.	B	\rightarrow Elim 2, 3
5.	B	Assp (\vee Elim)
6.	B	R5
7.	B	\vee Elim 1, 3–4, 5–6

(b) Prove that: C

1.	$A \vee B$	Pr.
2.	$(A \rightarrow C) \bullet (B \rightarrow C)$	Pr.
3.	A	Assp (\vee Elim)
4.	$A \rightarrow C$	\bullet Elim 2
5.	C	\rightarrow Elim 4, 3
6.	B	Assp (\vee Elim)
7.	$B \rightarrow C$	\bullet Elim 2
8.	C	\rightarrow Elim 7, 6
9.	C	\vee Elim 1, 3–5, 6–8

(c) Prove that: $D \vee G$

1.	$A \vee B$	Pr.
2.	$(A \vee C) \rightarrow D$	Pr.
3.	$G \equiv (\sim A \vee B)$	Pr.
4.	A	Assp (\vee Elim)
5.	$A \vee C$	\vee Int 4
6.	D	\rightarrow Elim 2, 5
7.	$D \vee G$	\vee Int 6
8.	B	Assp (\vee Elim)
9.	$\sim A \vee B$	\vee Int 8
10.	G	\equiv Elim 3, 9
11.	$D \vee G$	\vee Int 10
12.	$D \vee G$	\vee Elim 1, 4–7, 8–11

(d) Prove that: $H \bullet B$ (*version 1*)

1.	$A \bullet B$	Pr.	
2.	$A \rightarrow (G \vee H)$	Pr.	
3.	$G \equiv H$	Pr.	
4.	B	\bullet Elim 1	
5.	A	\bullet Elim 1	
6.	$G \vee H$	\rightarrow Elim 2, 5	
7.	G	Assp (\vee Elim)	
8.	H	\equiv Elim 3, 7	
9.	$H \bullet B$	\bullet Int 8, 4	
10.	H	Assp (\vee Elim)	
11.	$H \bullet B$	\bullet Int 10, 4	
12.	$H \bullet B$	\vee Elim 6, 7–9, 10–11	

(d) Prove that: $H \bullet B$ (*version 2*)

1.	$A \bullet B$	Pr.	
2.	$A \rightarrow (G \vee H)$	Pr.	
3.	$G \equiv H$	Pr.	
4.	B	\bullet Elim 1	
5.	A	\bullet Elim 1	
6.	$G \vee H$	\rightarrow Elim 2, 5	
7.	G	Assp (\vee Elim)	
8.	H	\equiv Elim 3, 7	
9.	H	Assp (\vee Elim)	
10.	H	R 10	
11.	H	\vee Elim 6, 7–8, 9–10	
12.	$H \bullet B$	\bullet Int 11, 4	

(e) Prove that: $G \bullet H$

1.	$A \bullet B$	Pr.	
2.	$A \rightarrow (G \vee H)$	Pr.	
3.	$G \equiv H$	Pr.	
4.	A	\bullet Elim 1	
5.	$G \vee H$	\rightarrow Elim 2, 4	
6.	G	Assp (\vee Elim)	
7.	H	\equiv Elim 3, 6	
8.	$G \bullet H$	\bullet Int 6, 7	
9.	H	Assp (\vee Elim)	
10.	G	\equiv Elim 3, 9	
11.	$G \bullet H$	\bullet Int 10, 9	
12.	$G \bullet H$	\vee Elim 5, 6–8, 9–11	

(f) Prove that: D

1.	$(B \vee A) \rightarrow D$	Pr.	
2.	$A \vee (B \vee C)$	Pr.	
3.	$C \equiv D$	Pr.	
4.	A	Assp (\vee Elim)	
5.	$B \vee A$	\vee Int 4	
6.	D	\rightarrow Elim 1, 5	
7.	$B \vee C$	Assp (\vee Elim)	
8.	B	Assp (\vee Elim)	
9.	$B \vee A$	\vee Int 8	
10.	D	\rightarrow Elim 1, 9	
11.	C	Assp (\vee Elim)	
12.	D	\equiv Elim 3, 11	
13.	D	\vee Elim 7, 8–10, 11–12	
14.	D	\vee Elim 2, 4–6, 7–13	

Exercises on Applying ~Int

~Int.I.a.

	p	$\sim p$
(a)	A	$\sim A$
(b)	$A \vee B$	$\sim(A \vee B)$
(c)	$\sim(A \cdot B)$	$\sim\sim(A \cdot B)$
(d)	$\sim A \vee B$	$\sim(\sim A \vee B)$
(e)	$\sim\sim A$	$\sim\sim\sim A$
(f)	$\sim B$	$\sim\sim B$

	p	$\sim p$
(g)	$C \cdot \sim D$	$\sim(C \cdot \sim D)$
(h)	$\sim A \equiv \sim(A \cdot B)$	$\sim(\sim A \equiv \sim(A \cdot B))$
(i)	$\sim(C \rightarrow B)$	$\sim\sim(C \rightarrow B)$
(j)	$\sim A \equiv C$	$\sim(\sim A \equiv C)$
(k)	$\sim\sim C$	$\sim\sim\sim C$
(l)	$\sim B \cdot B$	$\sim(\sim B \cdot B)$

~Int.I.b.

(a)

3	B	Assp (~Int)
5	$\sim C$	
8	C	
9	$\sim B$	~Int 3-5, 3-8

(b)

3	$A \vee B$	Assp (~Int)
5	$\sim C$	
8	$\sim\sim C$	
9	$\sim(A \vee B)$	~Int 3-5, 3-8

(c)

3	$\sim B$	Assp (~Int)
5	$\sim C \vee D$	
8	$\sim(\sim C \vee D)$	
9	$\sim\sim B$	~Int 3-5, 3-8

(d)

3	$\sim A \cdot \sim B$	Assp (~Int)
5	$\sim C$	
8	$\sim\sim C$	
9	$\sim(\sim A \cdot \sim B)$	~Int 3-5, 3-8

~Int.I.c.

(a)

3	C	Assp (~Int)
5	$\sim A$	
8	A	
9	$\sim C$	~Int 3-5, 3-8

(b)

3	$\sim A$	Assp (~Int)
5	$\sim B$	
8	B	
9	$\sim\sim A$	~Int 3-5, 3-8

(c)

3	$A \vee B$	Assp (~Int)
5	$\sim(C \cdot B)$	
8	$C \cdot B$	
9	$\sim(A \vee B)$	~Int 3-5, 3-8

(d)

3	$\sim(A \rightarrow B)$	Assp (~Int)
5	$\sim C$	
8	C	
9	$\sim\sim(A \rightarrow B)$	~Int 3-5, 3-8

~Int.II.

(a) Prove that: $\sim A$

1.	$A \rightarrow B$	Pr.
2.	$A \rightarrow \sim B$	Pr.
3.	A	Assp (\sim Int)
4.	B	\rightarrow Elim 1, 3
5.	$\sim B$	\rightarrow Elim 2, 3
6.	$\sim A$	\sim Int 3–4, 3–5

(b) Prove that: $\sim\sim C$

1.	$C \bullet B$	Pr.
2.	A	Pr.
3.	$\sim C$	Assp (\sim Int)
4.	C	\bullet Elim 1
5.	$\sim C$	R 3
6.	$\sim\sim C$	\sim Int 3–4, 3–5

(c) Prove that: $\sim(A \bullet B)$ [wersja A]

1.	$A \rightarrow C$	Pr.
2.	$B \rightarrow \sim\sim D$	Pr.
3.	$\sim C \bullet \sim D$	Pr.
4.	$A \bullet B$	Assp (\sim Int)
5.	A	\bullet Elim 4
6.	C	\rightarrow Elim 1, 5
7.	$\sim C$	\bullet Elim 3
8.	$\sim(A \bullet B)$	\sim Int 4–6, 4–7

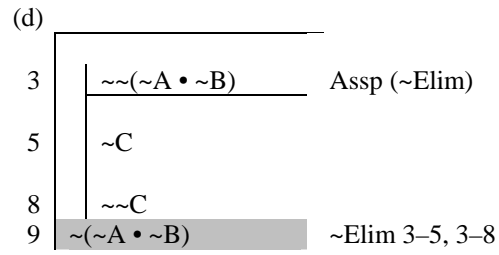
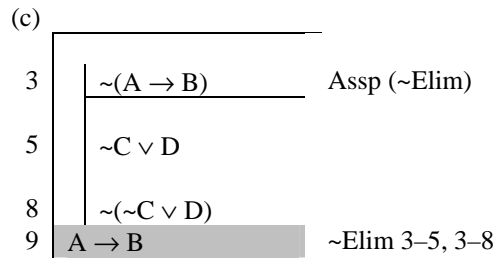
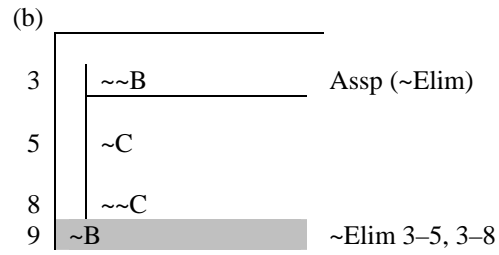
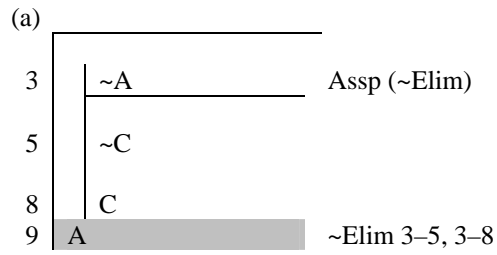
(c) Prove that: $\sim(A \bullet B)$ [wersja B]

1.	$A \rightarrow C$	Pr.
2.	$B \rightarrow \sim\sim D$	Pr.
3.	$\sim C \bullet \sim D$	Pr.
4.	$A \bullet B$	Assp (\sim Int)
5.	B	\bullet Elim 4
6.	$\sim\sim D$	\rightarrow Elim 2, 5
7.	$\sim D$	\bullet Elim 3
8.	$\sim(A \bullet B)$	\sim Int 4–6, 4–7

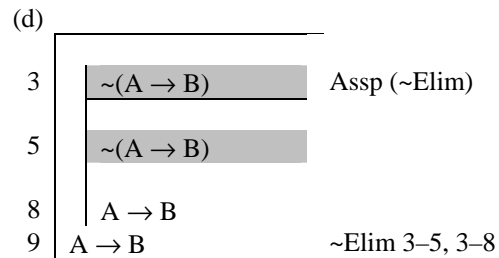
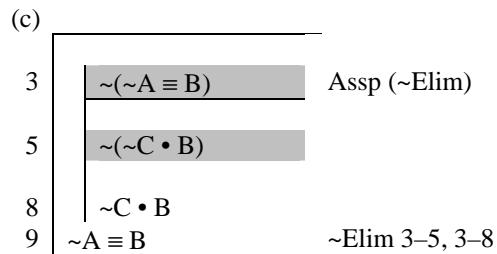
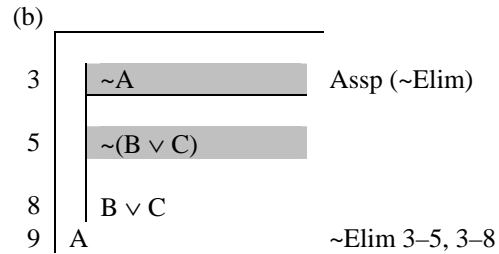
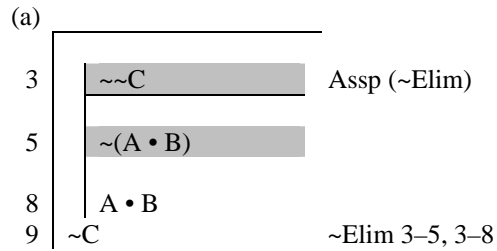
(d) Prove that: $\sim(A \equiv B)$

1.	$(A \equiv B) \rightarrow C$	Pr.
2.	$\sim(C \vee A)$	Pr.
3.	$A \equiv B$	Assp (\sim Int)
4.	C	\rightarrow Elim 1, 3
5.	$C \vee A$	\vee Int 4
6.	$\sim(C \vee A)$	R 2
7.	$\sim(A \equiv B)$	\sim Int 3–5, 3–6

~Elim.I.a.



~Elim.I.b.



~Elim.II.

(a) Prove that: A

1.	$\sim\sim A \cdot C$	Pr.												
2.	$B \cdot \sim\sim D$	Pr.												
<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="padding-right: 10px;">3.</td> <td style="border-left: 1px solid black; padding-left: 10px;">$\sim A$</td> <td style="padding-left: 20px;">Assp (~Elim)</td> </tr> <tr> <td>4.</td> <td style="border-left: 1px solid black; padding-left: 10px;">$\sim\sim A$</td> <td>•Elim 1</td> </tr> <tr> <td>5.</td> <td style="border-left: 1px solid black; padding-left: 10px;">$\sim A$</td> <td>R3</td> </tr> <tr> <td>6.</td> <td style="border-left: 1px solid black; padding-left: 10px;">A</td> <td>~Elim 3–4, 3–5</td> </tr> </table>			3.	$\sim A$	Assp (~Elim)	4.	$\sim\sim A$	•Elim 1	5.	$\sim A$	R3	6.	A	~Elim 3–4, 3–5
3.	$\sim A$	Assp (~Elim)												
4.	$\sim\sim A$	•Elim 1												
5.	$\sim A$	R3												
6.	A	~Elim 3–4, 3–5												

(b) Prove that: $\sim C$

1.	$\sim C \vee (B \vee \sim C)$	Pr.															
2.	$\sim B$	Pr.															
<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="padding-right: 10px;">3.</td> <td style="border-left: 1px solid black; padding-left: 10px;">$\sim\sim C$</td> <td style="padding-left: 20px;">Assp (~Elim)</td> </tr> <tr> <td>4.</td> <td style="border-left: 1px solid black; padding-left: 10px;">$B \vee \sim C$</td> <td>DS 1, 3</td> </tr> <tr> <td>5.</td> <td style="border-left: 1px solid black; padding-left: 10px;">B</td> <td>DS 3, 4</td> </tr> <tr> <td>6.</td> <td style="border-left: 1px solid black; padding-left: 10px;">$\sim B$</td> <td>R 2</td> </tr> <tr> <td>7.</td> <td style="border-left: 1px solid black; padding-left: 10px;">$\sim C$</td> <td>~Elim 3–5, 3–6</td> </tr> </table>			3.	$\sim\sim C$	Assp (~Elim)	4.	$B \vee \sim C$	DS 1, 3	5.	B	DS 3, 4	6.	$\sim B$	R 2	7.	$\sim C$	~Elim 3–5, 3–6
3.	$\sim\sim C$	Assp (~Elim)															
4.	$B \vee \sim C$	DS 1, 3															
5.	B	DS 3, 4															
6.	$\sim B$	R 2															
7.	$\sim C$	~Elim 3–5, 3–6															

(c) Prove that: $A \cdot B$

1.	$\sim(A \cdot B) \rightarrow (C \cdot D)$	Pr.															
2.	$\sim D \cdot \sim B$	Pr.															
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3.	$\sim(A \cdot B)$	Assp (~Elim)															
4.	$C \cdot D$	\rightarrow Elim 1, 3															
5.	D	•Elim 4															
6.	$\sim D$	•Elim 2															
7.	$A \cdot B$	~Elim 3–5, 3–6															

(d) Prove that: C

1.	$\sim C \rightarrow \sim B$	Pr.															
2.	$\sim A \cdot (\sim B \rightarrow B)$	Pr.															
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