Workbook Unit 10:

Natural Deduction Proofs (A)

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drp@swps.edu.pl

Overview

This unit introduces

- a new method of showing that an argument is valid

We will still be working with the concept of validity but we will be using a method of checking validity that is closer to ordinary reasoning (though we will work in symbols). The deductive method of checking validity is in many ways easier and more intuitive than the truth-table method. However, it will be more difficult at first. Prepare to do all the exercises – you need to become really familiar with the "moves" in order to be able to use the method.

It is best to proceed gradually. I have divided the rules in a way that (I hope) will make it easier for you to learn the proof method gradually. It is crucial that you do the exercises after you have learned a couple of the rules. Moreover, we will first do simpler exercises and only later work with the more difficult ones. Remember, however, that you need to practice a lot.

This unit

- introduces the general idea of natural deduction
- introduces three inference rules: •Int, •Elim., →Elim
- teaches you how to apply inference rules correctly
- teaches you how to construct relatively simple proofs using the three rules.

PowerPoint Presentation

There is a PowerPoint Presentation that accompanies this Unit. It is available on-line as a .pps and a zipped .pps file.

Prerequisites

There are is only one prerequisite for this unit. You need to be able to symbolize English sentences (Units 2, 3).

0. How to Learn Proofs

Perhaps the most important thing for you to do for this unit is to do the in-text exercises that are listed here after almost every little section. Moreover, you should complete them before you go on to the next section! After you completed them, you should check that your solutions are correct against the solutions I provide. If some of your answers are incorrect, you need to figure out what went wrong. Reread the section, look at other examples, if you can't figure out what went wrong, ask someone (either me or raise the question on the Bulletin Board).

I do hope that you will find those exercises fun. They should be kind of like puzzles for you. They are going to strain you a little. But if you do them systematically and *at the time at which you are told to do them*, you will actually enjoy them.

The worst thing that you can do for this unit is to think that you should just read through the whole material first and then do a couple of the exercises or take the quiz. You will fail, I guarantee. We have reached this point in the course, where 90% of students fail unless they prepare *as I say!* Trust me, I've had experience with this.

This is the point of the course where you must think about this Workbook as a diet book. Just as reading a "How to get thin" book will not make you slimmer, so *reading* "How to do proofs" will not teach you proofs. In both cases, you need to *apply* what you are reading!

1. About Natural Deduction Proofs – General Comments

Consider the following reasoning:

> If you pass logic, your best friend will invite you out to dinner in either a French or an Italian restaurant. If you study hard but also watch a lot of TV then he will not invite you to a French restaurant. You will pass logic if you work hard. You certainly did work hard though you also watched a lot of TV. So…

If you completed the ellipsis with "your friend invited you to an Italian restaurant" you did more than guess. You *followed* the reasoning, which means that you did a little proof in your head.

Let's try to represent the story in a semi-proof form. We start out by listing what we know:

(1)	If you pass logic, your best friend will invite you out to dinner in either a French or an Italian restaurant.	Pr.
(2)	He will not invite you to a French restaurant if you study hard but also watch a lot of TV.	Pr.
(3)	You will pass logic if you work hard.	Pr.
(4)	You certainly did work hard though you also watched a lot of TV.	Pr.

Since these are the items that we are given, we will mark them as premises ('Pr.' is short for 'premise'). We number them so that it is easier to refer to them.

Now, let's proceed to find out what we can figure out from the premises, step by step. (Reconstructing the reasoning step by step helps us to keep track of our reasoning – we can always check whether the logical step we've made is correct or not.) One thing that would be good to figure out, is whether you will pass logic: premise (3) tells us that you will pass logic if you work hard, while (4) tells us that you did work hard and also watched a lot of TV. Premises (3) and (4) tell us enough so that we can safely say that you will pass logic (you will later see that we will be breaking this reasoning into two steps but for now we are just trying to get hang on the sort of thing we will be doing):

(5)	You will pass logic	(3), (4)

(Note that on the right hand side, we have put the numbers of the statements that were necessary to justify our drawing the inference that you will pass logic.) Now, we are ready to draw the inference that your best friend will invite you out to dinner in either a French or an Italian restaurant. We are justified in drawing this inference because we now know that you will pass logic (5) and we knew already that your best friend will invite you out to dinner in either a French or an Italian restaurant if you pass logic (1):

(6)	Your best friend will invite you out to dinner in either a French or an Italian restaurant	(1), (5)

Now we need to find out to which restaurant you will go. If we look at statement (2) it tells you the conditions under which you will not be invited to a French restaurant, and since those conditions are in fact met (4), we know that:

(7) Your best friend will not invite you out to a French restaurant (2), (4)

Now, it is easy to see that your friend will invite you to an Italian restaurant – it was a choice between French and Italian, and because you watched a lot of TV, he does not see it fit to invite you to a French restaurant, so:

(8) Your best friend will invite you out to an Italian restaurant (7), (6)

In the above example, the semi-proof might seem like much ado about not very much since we were quite capable of finding the conclusion without actually explicitly going step-by-step. However, we are not always capable of reasoning out the conclusion as easily. Consider the following example:

> Tommy is not interested in going out with Jane; Quentin, on the other hand, is quite interested in going out with Jane. Jane will go out with Quentin if either Quentin makes it to the football team or Gary is not be interested in going out with Jane. Quentin will make the football team if either Susan does not go out with him or Bob does not make the team. Susan will not go out with Quentin unless Jane invites her to her party. If Tommy is not interested in going out with Jane, she will invite neither him nor Susan to her party. So …

Here it is a little harder to see what the conclusion is. Will Jane go out with Quentin? The answer is yes, but in order to see that it indeed follows from the story, we need to use some more systematic method for seeing what follows from what. We need to construct a semi-proof.

Let's try to do the same thing with the second reasoning. Here, I will write out the reasoning marking the steps and not commenting on them. You should check that the steps in fact follow by inserting the appropriate justifications, i.e. the lines of the statements used to justify a given step (The answer follows below.)

(1)	Tommy is not interested in going out with Jane; Quentin, on the other hand, is quite interested in going out with Jane.	Pr.
(2)	Jane will go out with Quentin if either Quentin makes it to the football team or Gary is not be interested in going out with Jane.	Pr.
(3)	Quentin will make the football team if either Susan does not go out with him or Bob does not make the team.	Pr.
(4)	Susan will not go out with Quentin unless Jane invites her to her party.	Pr.
(5)	If Tommy is not interested in going out with Jane, she will invite neither him nor Susan to her party.	Pr.
(6)	Jane will invite neither Tommy nor Susan to her party.	
(7)	Susan will not go out with Quentin	
(8)	Quentin will make the football team	
(9)	Jane will go out with Quentin	

derivation line

premise line

Answer: Line 6: (1), (5); line 7: (4), (6); line 8: (3), (7); line 9: (2), (8).

If you have understood the above, you have already understood essentially what proofs are – they are just sequences of statements such that every new statement follows from some of the statements that precede it. In logic, however, we need to be more systematic about what follows from what. This process is governed by inference rules which tell us what we are allowed to infer from what.

The premise line and the derivation line are graphical markers – drawn sometimes to help you visualize the derivation process. They are not necessary, however.

Inference Rules

I said that inference rules govern what can be inferred from what. The rules of inference are simply valid reasoning patterns. We will introduce nine inference rules – three in this unit, and the remaining ones in the next two units.

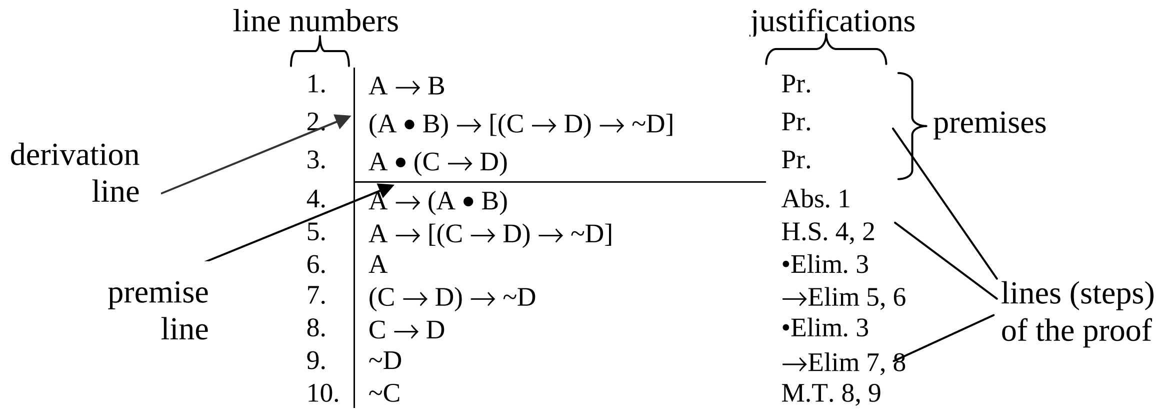
You not only need to learn all the nine rules of inference by heart. They need to become your second nature, as it were. You need to learn them in such a way that when you are confronted with a proof setting, you will just *see* which rules you can apply. The rules need to literally jump at you (in your head). The only way to reach this kind of competence with the rules is to do the exercises. If you don't, you will be lost with proofs.

The trickiest part of learning proofs is learning to apply inference rules. One thing that is tricky is that the process of applying inference rules is strictly determined by the logical form and most of our minds are just not geared to such precision. But there is no way around this. You need to learn to look at the statements like a computer would. I've provided a lot of exercises in the book. You should do all those exercises and immediately after you do them, check the results. If you make mistakes, you need to understand why. Only when you are able to do all the exercises correctly can you proceed.

My main advice is not to wait. The more often you train, the better your logic muscles will become.

2. What does a Natural Deduction Proof Look Like?

Like this, for example (but you should not even try to understand the proof – you will be able to grasp it at the end of the third unit on proofs; for now I'm using it just to show you what a proof looks like):



A proof is a sequence of steps, which we also call "lines". Each step of the proof (each line of the proof) consists of a number (which helps us refer to the line), a statement contained on the line, and its justification. All three elements are essential. The justification of a statement can be either 'Pr.', which means that the statement is a premise, or it cites one of the inference rules we will be learning and numbers of the lines to which the inference rule was applied (each of the inference rules has a precise format in which the justification needs to be given; this is also something you will be learning).

The last line of a proof contains the conclusion that was to be derived.

The derivation line and the premise line are graphical markers that help us to keep our eye on the proof. They are of graphical help only. You will in particular not use them on-line. But they are helpful devices and I use them throughout the exercises.

3. Conjunction, Simplification and *Modus Ponens*

We will start with three simple and intuitive inference rules. You should follow the order in which I introduce them and do the exercises at the time you are asked. There are two things you need to learn to do: (1) be able apply the rules, (2) be able to construct proofs using them.

3.1. Conjunction Introduction (•Int.)

Line *i*.	*p*	
Line *j*.	*q*	
∴	***p* • *q***	**•Int. *i*, *j***

The rule tells you: if you have two statement forms (however complex) *p* and *q*, you are allowed to infer their conjunction (*p* • *q*).

Conjunction is a very intuitive rule. If you know that both *p* and *q* are true then their conjunction must be true too.

Note about convention: I am marking what you are allowed to infer in **green, bold and big**, and what needs to be given in order for you to infer the conclusion in blue. It is crucial for you to mark this difference.

> Since you are probably working with a black and white printout, you should reach for your school gear and mark these statements in appropriate color. Don't underestimate the importance of coloring!!! It will help you keep track of what you can infer and what you must have as a given – failure to note this is one of the most common mistakes in applying the third rule →Elim

Inference rules tell you what you can infer on the basis of what you already have. The proof line where you infer the conclusion needs to be justified according to the format given where *i* and *j* are replaced by the numbers of the lines with the statements needed to infer the conclusion.

Illustrations

J	Betty promised Jack that she will go out with him tonight.
T	Betty promised Tim that she will go out with him tonight.
J • T	So, Betty promised both Jack and Tim that she will go out with them tonight.

R → M	If it rains, Jane will go to the mall.
S → P	If it is sunny, Jane will go to the park.
(R → M) • (S → P)	So, Jane will go to the mall if it rains, and she will go to the park if it's sunny.

Do note that *p* and *q* can be very complex.

Common Mistakes

One of the notorious mistakes about Conjunction is to think that it applies for other connectives too. This is wrong. Conjunction holds only for the dot. You cannot infer any other type of statement in this way.

	1.	A	Pr.
	2.	B	
✖	~~3.~~	~~A ∨ B~~	~~•Int. 1, 2~~

	1.	A	Pr.
	2.	B	
✖	~~3.~~	~~A → B~~	~~•Int. 1, 2~~

	4.	A	Pr.
	5.	B	
✖	~~6.~~	~~A ≡ B~~	~~•Int. 1, 2~~

Exercises on Applying •Int.

Check your answers with *Solutions*. Do these exercises now!

•Int.I. Please, fill in the missing information:

1.	C	Pr.
2.	D	Pr.
3.	C • D	
4.	D • C	

1.	A	Pr.
2.	B	Pr.
3.		•Int. 1, 2
4.		•Int. 2, 1

1.		Pr.
2.		Pr.
3.	B • C	•Int. 1, 2
4.		•Int. 2, 1

1.	~C	Pr.
2.	B	Pr.
3.	~C • B	
4.		•Int. 2, 1

1.	C	Pr.
2.	A ≡ B	Pr.
3.		•Int. 1, 2
4.		•Int. 2, 1

1.	C • A	Pr.
2.	B	Pr.
3.		•Int. 1, 2
4.		•Int. 2, 1

1.	~A	Pr.
2.	~~B	Pr.
3.		•Int. 1, 2
4.		•Int. 2, 1

1.		Pr.
2.		Pr.
3.	(C ∨ A) • D	•Int. 1, 2
4.		•Int. 2, 1

1.		Pr.
2.		Pr.
3.	~B • (C → D)	•Int. 1, 2
4.		•Int. 2, 1

3.2. Simplification (•Elim.)

Line *i*.		$p \bullet q$	
	∴	p	**•Elim. *i***
Line *i*.		$p \bullet q$	
	∴	q	**•Elim. *i***

The rule tells you: if you have a conjunction, you are allowed to infer one of its conjuncts.

Simplification is one of the most obvious deductive rules. If you know that a conjunction of two statements is true then you also know that its conjunct must be true.

There are two rules by the same name. The first allows you to infer the first conjunct. The second allows you to infer the second conjunct.

Illustration

F • R	Jim will get an F in English but he will regret it.
F	So, Jim will get an F in English.
L • (S → H)	Mark will fail logic but if Susie loves him, he will be happy.
S → H	So, if Susie loves Mark, he will be happy.

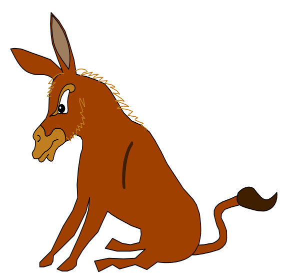
Common Mistakes

One of the notorious mistakes about Simplification is to think that it applies to other types of statements too. This is wrong. You can apply Simplification only to conjunctions. You cannot simplify any other type of statement in this way. So:

1. A ∨ B Pr.
✖ ~~2. A •Elim. 1~~

1. A → B Pr.
✖ ~~2. A •Elim. 1~~

1. A ≡ B Pr.
✖ ~~2. A •Elim. 1~~



Another common mistake is to think that you **•Elim** "sees into" the parentheses (see also §0):

1. (A • B) • C Pr.
✖ ~~2. A •Elim. 1~~

•Elim. tells you that that you are allowed to infer one of the conjuncts, but (A • B) • C has exactly two conjuncts: the first conjunct is (A • B) and the second conjunct is C. Since A is neither the first nor the second conjunct of (A • B) • C, it cannot be inferred by means of •Elim. Not in one step. But of course we can derive A in two steps, like this:

1.	(A • B) • C	Pr.
2.	A • B	•Elim. 1
3.	A	•Elim. 2

Note that the justification for line 3 refers not to line 1 (that was the error above), but to line 2. Everything is fine now. On line 2 we have a conjunction A • B, whose first conjunct is A. We can thus safely apply •Elim. to line 2.

Exercises on Applying •Elim.

Check your answers with *Solutions*. DO NOT postpone doing these exercises. Do them now!

•Elim.I. Please, fill in the missing information:

1.	A • B	Pr.
2.	A	
3.	B	

1.	C • D	Pr.
2.		•Elim. 1
3.		•Elim. 1

1.		Pr.
2.	A	•Elim. 1
3.	C	•Elim. 1

1.	C • B	Pr.
2.	B ≡ D	Pr.
3.	B	

1.	C ∨ A	Pr.
2.	A • B	Pr.
3.	A	

1.	(C ∨ A) • D	Pr.
2.	D → B	Pr.
3.	D	

1.	~A • C	Pr.
2.	B • ~D	Pr.
3.		•Elim. 1
4.		•Elim. 1
5.		•Elim. 2
6.		•Elim. 2

1.	A • (C ∨ B)	Pr.
2.	(B ≡ D) • D	Pr.
3.		•Elim. 2
4.		•Elim. 2
5.		•Elim. 1
6.		•Elim. 1

1.	(A → C) • ~D	Pr.
2.	(B • A) • C	Pr.
3.		•Elim. 1
4.		•Elim. 1
5.		•Elim. 2
6.		•Elim. 2

3.3. Proofs Using •Int and •Elim

Now that you are able to apply the •Int and •Elim inference rules, let's use them in some simple proofs. We will do so on examples. Ideally, you should try to work out the proof on your own and then read and compare your attempt with what is said here. Toward the end of this unit, you will have plenty step-by-step exercises to practice on.

Example 1

Using the proof method, show that the following argument is valid:

A
B
C
―――――
(A • B) • C

We start by listing, numbering and justifying the premises (it is also useful to put on the side the conclusion, i.e. the statement that we are supposed to prove, so that we know in what order to proceed; I've put it after the '∴' sign):

1. A	Pr.	∴ (A • B) • C
2. B	Pr.	
3. C	Pr.	

Let's think about strategy. We are supposed to obtain a conjunction, whose first conjunct is complex (indeed it is itself a conjunction, i.e. A • B) and whose second conjunct is C. We could apply the **•Int.** Rule if we had both conjuncts on their own lines. We do have C, but we do not have A • B. Nothing lost, we need to figure out whether we could not obtain A • B from what we have. And the answer is that we can, if we apply **•Int** to lines 1 and 2. Let's write in the statement and its justification.

4. A • B	•Int. 1, 2

Now, we can apply **•Int.** again to line 4 and 3 and obtain the statement wanted:

5. (A • B) • C	•Int. 3, 4

Try to do the proof without looking above:

1.	A	Pr.
2.	B	Pr.
3.	C	Pr.

Example 2

Prove that A • (B • C) from the following three premises – you should try to do the proof yourself and then check against the solution provided here:

1.	A	Pr.
2.	B	Pr.
3.	C	Pr.

Let's think about strategy. We are supposed to obtain a conjunction, whose first conjunct is A and whose second conjunct is complex (indeed it is itself a conjunction, i.e. B • C). We could apply the **•Int.** rule if we had both conjuncts on their own lines. We do have A, but we do not have B • C. However, we can obtain B • C if if we apply **•Int** to lines 2 and 3. Let's write in the statement and its justification.

4. B • C •Int. 2, 3

Now, we can apply **•Int.** again to line 1 and 4 and obtain the statement wanted:

5. A • (B • C) •Int. 1, 4

Example 3

Prove that [A • (B • ~C)] • (A • ~C) from the following premises:

1.	A	Pr.
2.	B	Pr.
3.	~C	Pr.

This is a more complex case, but if you have understood the above, you will have no problem doing it. In any case, you should try doing the proof without looking at the solution that follows.

Let's try to think about the strategy first. We need a conjunction, whose first conjunct is complex, viz. A • (B • ~C), and whose second conjunct is also complex, viz. A • ~C. We cannot apply **•Int** yet because none of these statements is on any of the first three lines that we have. But we can certainly obtain them. How do we obtain A • (B • ~C)? It is a conjunction – we have the first conjunct, A, but we don't have the second conjunct, B • ~C. Can we obtain B • ~C? Yes, if we apply •Int. to lines 2 and 3. So, let's do it:

4. B • ~C •Int. 2, 3

Now we can obtain the first conjunct of the big conjunction we are trying to prove:

5. A • (B • ~C) •Int. 1, 4

We have thus completed the first half of our task. We need to complete the second by proving A • ~C. How? You guessed it, by applying •Int. to lines 1 and 3:

6. A • ~C •Int. 1, 3

Now we have the first and the second conjunct of our conclusion on their own lines, so we can conjoin them:

7. [A • (B • ~C)] • (A • ~C) •Int. 5, 6

Example 4

Prove that A • B from the following premises:

1.	A • C	Pr.
2.	B	Pr.

Let's try to think about the strategy first. We need a conjunction, whose first conjunct is A and the second conjunct is B. We could obtain it using **•Int** if we had both conjuncts on their own lines – but we only have B on its own line, we don't have A on its own line. Can we get A to stand on its own line? The answer is yes, if we apply **•Elim** to line 1. Thus:

3. A •Elim. 1
4. A • B •Int. 2, 3

Example 5

Prove that (B → A) • B from the following premises:

1.	B • C	Pr.
2.	B → A	Pr.

Let's try to think about the strategy first. We need a conjunction, whose first conjunct is B → A and the second conjunct is B. We could obtain it using **•Int** if we had both conjuncts on their own lines – but we only have B → A on its own line, we don't have B on its own line. Can we get B to stand on its own line? The answer is yes, if we apply **•Elim** to line 1. Thus:

3. B •Elim. 1

Please, note that we cannot obtain B from line 2! **•Elim** applies only to conjunctions!

4. (B → A) • B •Int. 2, 3

Exercises on Proofs with •Elim. and •Int.

Check your answers with *Solutions*. DO NOT postpone doing these exercises. Do them now!

•Int-•Elim.II. The following proofs are missing exactly one step to prove the conclusion (on the last line). Please, fill in the missing step, justify it and justify the last step:

1.	A	Pr.
2.	B	Pr.
3.		
4.	(A • B) • B	

1.	A	Pr.
2.	B	Pr.
3.		
4.	A • (B • A)	

1.	A	Pr.
2.	B	Pr.
3.		
4.	B • (A • B)	

1.	C • A	Pr.
2.	B	Pr.
3.		
4.	A • B	

1.	C	Pr.
2.	B	Pr.
3.		
4.	(B • C) • B	

1.	C	Pr.
2.	A • B	Pr.
3.		
4.	A • C	

1.	~C	Pr.
2.	~D	Pr.
3.		
4.	(~C • ~D) • ~C	

1.	C • ~D	Pr.
2.	A → B	Pr.
3.		
4.	~D • (A → B)	

1.	(A • C) • D	Pr.
2.		
3.	A	

•Int-•Elim.III. The following proofs are missing exactly two steps to prove the conclusion (on the last line). Please, fill in the missing steps, justify them and justify the last step.

1.	A • B	Pr.
2.	C • D	Pr.
3.		
4.		
5.	A • D	

1.	(A • B) • C	Pr.
2.	D	Pr.
3.		
4.		
5.	A • D	

1.	C • [A • (B • D)]	Pr.
2.		
3.		
4.	B	

1.	C • A	Pr.
2.		
3.		
4.	A • C	

1.	(A • B) • (C ∨ D)	Pr.
2.	~G	Pr.
3.		
4.		
5.	~G • B	

1.	A	Pr.
2.	B	Pr.
3.		
4.		
5.	(A • B) • (B • A)	

•Int-•Elim.IV. Prove that the indicated conclusion follows from the premises given (you need to determine how many steps are necessary to prove the conclusion):

a. Prove that (A • C) • (C • A)

1.	A	Pr.
2.	C	Pr.

b. Prove that (A • C) • (C • A)

1.	A • B	Pr.
2.	B • C	Pr.

c. Prove that (B • C) • (A ≡D)

1.	(A • B) • ~C	Pr.
2.	(C • D) • (A ≡ D)	Pr.

d. Prove that A • [B • (A • C)]

1.	(A • B) • C	Pr.

3.4. Modus Ponens (→Elim)

Line *i*.	$p \to q$		
Line *j*.	p		
	∴	$\boldsymbol{q}$	**→Elim i, j**

The rule tells you: if you have two statement forms: a conditional ($p \to q$) and its antecedent (p), you are allowed to infer the consequent (q).

Illustrations

C → H
C
———
H

If Susie is a good **c**ook, her mother-in-law will **h**ate her.
Susie is a good **c**ook
So, her mother-in-law is bound to **h**ate her.

R → (T ∨ M)
R
———
T ∨ M

If it **r**ains, Sunny will go either to the **t**heatre or to the **m**ovies.
Unfortunately it **r**ains.
So, Sunny will go either to the **t**heatre or to the **m**ovies.

Common Mistakes

One of the notorious mistakes to avoid is the failure to realize what →Elim requires that you have given and what it allows you to infer. Some students provide the following sequence, mistakenly thinking that it is deductive reasoning:

1.	A → B	Pr.
✖ ~~2.~~	~~A~~	~~→Elim 1~~
3.	B	→Elim 1, 2

What is wrong here? Surely, step 3 is justified by →Elim. However, Step 2 is not justified by →Elim! If you are given the conditional A → B, you cannot infer that A. Why not? Well suppose I argue like this: "If you owe me $100, you should pay me $100 back. So, you owe me $100!" You should be alarmed! (I could easily go higher in my financial aspirations…) Fortunately the conclusion does not follow.

Just to be sure, let me present you with a valid argument along these lines. "If you owe me $100, you should pay me $100 back. In fact, you do owe me $100. So, you should pay me back $100." The structure of this argument is:

1.	A → B	Pr.
2.	A	Pr
3.	B	→Elim 1, 2

The difference between the wrong reasoning and the correct reasoning just above is that Premise 2 here is not attempted to be justified by →Elim It is a premise – you *do* need it *given* (as a premise or justified by another rule).

Modus Ponens SAP

The "Modus Ponens" SAP (short animated presentation) is available as a PowerPoint show on-line. It should help you visualize what →Elim is about.

Exercise on Applying →Elim

Check your answers with *Solutions*. DO NOT postpone doing these exercises. Do them now!

→**Elim-I.** Please, fill in the missing information:

1.	C → D	Pr.
2.	C	Pr.
3.	D	

1.	A → D	Pr.
2.	A	Pr.
3.		→Elim 1, 2

1.	B → D	Pr.
2.		Pr.
3.	D	→Elim 1, 2

1.	D → A	Pr.
2.	D	Pr.
3.		→Elim 1, 2

1.		Pr.
2.	C	Pr.
3.	D	→Elim 1, 2

1.		Pr.
2.	D	Pr.
3.	C	→Elim 1, 2

1.	~B → A	Pr.
2.	~B	Pr.
3.		→Elim 1, 2

1.		Pr.
2.	~A	Pr.
3.	B	→Elim 1, 2

1.	A → ~B	Pr.
2.		Pr.
3.	~B	→Elim 1, 2

1.		Pr.
2.	~C	Pr.
3.	D	→Elim 1, 2

1.	~A → ~D	Pr.
2.	~A	Pr.
3.		→Elim 1, 2

1.	B → ~B	Pr.
2.	B	Pr.
3.		→Elim 1, 2

1.	B	Pr.
2.	B → ~D	Pr.
3.		→Elim 1, 2

1.		Pr.
2.	D → B	Pr.
3.		→Elim 1, 2

1.	~~A	Pr.
2.		Pr.
3.	C	→Elim 1, 2

3.5. Proofs Using →Elim

Example 1

Prove that C from the following premises:

1.	A • B	Pr.
2.	A → C	Pr.

Think about strategy. You are to obtain C on its own. The only place where you can find C is in the second premise – C is the consequent of a conditional. There is a rule of inference which allows you to infer the consequent of a conditional – but only if you have the antecedent (here: A) standing on its own line. You don't have A on its own. Can you get it? Yes, by applying •Elim. to line 1. Thus:

3. A	•Elim. 1
4. C	→Elim 2, 3

Example 2

Prove that B • C from the following premises:

1.	A	Pr.
2.	A → C	Pr.
3.	B	Pr.

The argument is valid – the conclusion (B • C) follows from the premises. You can check it using truth tables but it is much easier to establish this using our inference rules. Try to do the proof on your own and later check it against the solution provided here. I'll go through the steps in more detail trying to systematize and make explicit what you have been doing in doing proofs:

STEP 1: ***Preparation for the deduction.*** You should list and number the premises, as well as mark (on the right) that they are premises. It is a good idea to write somewhere near the deduction what the conclusion you are trying to derive is.

STEP 2: ***Think about Strategy*** There are two strategies you can use in a derivation – working forward and working backward. It is useful to think which to apply, and it will be sometimes possible to apply both.

Working Forward. The strategy basically consists in figuring out what inference rules you *can* at all apply to what you have. Here, for instance, the application of →Elim invites itself on the first two premises, so:

4. C →Elim 1, 2

This is a "lucky" step because it then becomes quite obvious that we can also obtain the conclusion by conjoining statements in lines 3 and 4 thus:

5. B • C •Int. 3, 4

However, while you should always be able to use the strategy of working forward, you should also be weary: at any point in time you can apply a lot of rules to what you have, which will lead you nowhere near the conclusion. (And occasionally – in more difficult proofs, there will be irrelevant premises that you should ignore rather than use.) This is why you should always have the conclusion in view and try to use the strategy of working backward – at least in part.

Working Backward. The strategy of working backward essentially consists in your asking yourself *how to get the conclusion* from what you have. With the more complex proofs you might subdivide your task of deriving the conclusion by asking yourself what intermediate statements (subconclusions) you would need in order to prove the conclusion. Here the reasoning is relatively simple. When presented with the premises and the conclusion you are to derive you would reason like this: "I need to derive a conjunction B • C. I could derive B • C using **•Int.** if I had both conjuncts on separate lines. I already have B, but I don't have C. Can I get C to stand on a separate line? Yes, if I apply →**Elim** to lines 1 and 2. Hence the proof will be completed by adding the following lines:

4. C →Elim 1, 2
5. B • C •Int. 3, 4

In the following examples, I will be focusing on the strategy of working backward. But you should feel free to use both. Remember, however, that you often cannot complete derivations unless you *also* use some working backward.

Exercises on Proofs with $\to$Elim

Check your answers with *Solutions*. DO NOT postpone doing these exercises. Do them now!

$\to$Elim-II. The following proofs are missing exactly one step to prove the conclusion (on the last line). Please, fill in the missing step, justify it and justify the last step:

1.	$B \bullet C$	Pr.
2.	$B \to A$	Pr.
3.		
4.	A	

1.	C	Pr.
2.	$(C \to B) \bullet A$	Pr.
3.		
4.	B	

1.	$(B \to A) \bullet C$	Pr.
2.	B	Pr.
3.		
4.	A	

1.	$A \to D$	Pr.
2.	$D \to C$	Pr.
3.	A	Pr.
4.		
5.	C	

1.	$B \to C$	Pr.
2.	$A \to B$	Pr.
3.	A	Pr.
4.		
5.	C	

1.	$B \to \sim C$	Pr.
2.	$\sim A \to B$	Pr.
3.	$\sim A$	Pr.
4.		
5.	$\sim C$	

1.	A	Pr.
2.	B	Pr.
3.	$(A \bullet B) \to C$	Pr.
4.		
5.	C	

1.	$B \to (C \bullet A)$	Pr.
2.	$B \bullet D$	Pr.
3.		
4.	$C \bullet A$	

1.	$B \to (B \to A)$	Pr.
2.	B	Pr.
3.		
4.	A	

$\to$Elim-III. The following proofs are missing exactly two steps to prove the conclusion (on the last line). Please, fill in the missing steps, justify them and justify the last step:

1.	$(A \to B) \bullet C$	Pr.
2.	$C \bullet A$	Pr.
3.		
4.		
5.	B	

1.	$A \to B$	Pr.
2.	$C \bullet (A \bullet D)$	Pr.
3.		
4.		
5.	B	

1.	$B \to C$	Pr.
2.	$C \to D$	Pr.
3.	$A \bullet B$	Pr.
4.		
5.		
6.	D	

1.	$B \to C$	Pr.
2.	$A \to B$	Pr.
3.	A	Pr.
4.		
5.		
6.	$B \bullet C$	

1.	$C \to A$	Pr.
2.	$(A \to C) \bullet C$	Pr.
3.	$A \to B$	Pr.
4.		
5.		
6.	B	

1.	$\sim D \to B$	Pr.
2.	$\sim A \to \sim D$	Pr.
3.	$\sim A$	Pr.
4.		
5.		
6.	$\sim D \bullet B$	

1.	C	Pr.
2.	$C \to B$	Pr.
3.	$C \to D$	Pr.
4.		
5.		
6.	$B \bullet D$	

1.	$C \bullet A$	Pr.
2.	$A \to (D \bullet C)$	Pr.
3.		
4.		
5.	D	

1.	$\sim D \bullet A$	Pr.
2.	$(\sim D \bullet B) \to C$	Pr.
3.	B	Pr.
4.		
5.		
6.	C	

→Elim-IV. Prove that the indicated conclusion follows from the premises given:

Prove that A • C from:

1.	(A → B) • (B → C)	Pr.
2.	A	Pr.

Prove that B • D from:

1.	A → B	Pr.
2.	C → D	Pr.
3.	A • C	Pr.

4. Inference Rules do Not Apply to Statement Components

One very important point about inference rules is that

Inference rules "see" only the main connectives
Inference rules don't apply to statement components

4.1. "Inference Rules do Not Apply to Statement Components!" What does It Mean?

Example 1.

Suppose that you are given the following two statements:

(A → C) ∨ B
A

Can you apply Modus Ponens (→Elim) here?

If you have answered 'yes', you have committed one of the most frequent – and dreadful – errors of proving.

(A → C) ∨ B
A
✖ ~~∴C →Elim~~

In fact, you cannot apply →Elim here: the first statement is a conjunction – it is not a conditional! You can apply Modus Ponens only if the main connective of one of the statements is the horseshoe.

This is what it means to say that inference rules don't apply to statement components.

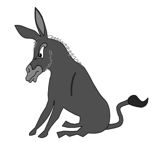
Example 2.

Suppose that you are given the following two statements:

$A \rightarrow C$
$A \vee B$

Can you apply Modus Ponens?

Again, if you answered ‘yes’, you committed an error:



$A \rightarrow C$
$A \vee B$
✖ ~~∴C →Elim~~

Although now one of the statements is a conditional – the other is not the antecedent of the conditional (though it contains the antecedent of the conditional as a component, this is not a circumstance where you are allowed to apply the inference rules). You could only apply →Elim here if A was standing on its own.

Example 3.

Suppose that you are given the following statement:

$(A \bullet C) \bullet B$

Can you apply Simplification (in one step) to get A?

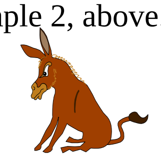
$(A \bullet C) \bullet B$
✖ ~~∴A •Elim.~~

No!!!! A is the conjunct of a component of the statement! You could eventually “retrieve” A by applying simplification but you need to apply it twice – like this:

1. $(A \bullet C) \bullet B$	Pr.
2. $A \bullet C$	•Elim. 1
3. A	•Elim. 2

4.2. Why do Inference Rules Not Apply to Statement Components?

If you look at Example 3 especially this property of inference rules, viz. the fact that they do not apply to statement components, will seem just tiresome. But there is good reason. The whole idea behind our ability to construct proofs is to provide a perfectly general method for deriving some statements from others, where the derivations are supposed to be valid, i.e. truth-preserving (if you start with truth you cannot get anything but the truth). And while there are some cases (like in Example 3) where we would be able to derive the wanted statement from the premises any way, there will be others where such a derivation is impossible. Consider Example 2, above.



$A \rightarrow C$
$A \vee B$
✖ ~~∴C →Elim~~

This argument is just invalid. You can see this by assigning appropriate truth-values to the simple statements, but it will be perhaps even more vivid if you consider an argument sharing the same argument form, where A stands for ‘You owe me

$1,000,000.00”, C stands for ‘You should pay me $1,000,000.00’, and B stands for ‘You are taking PHI 253’. The relevant substitution instance will have true premises, but – you should feel lucky! – a false conclusion:

> If you owe me $1,000,000.00, then you should pay me $1,000,000.00 [true]
> Either it is true that you owe me $1,000,000.00 or it is true that
> you are taking PHI 253 [true]
> So, you should pay me $1,000,000.00 [false]

Think about this for a minute! The reason why you should not be paying me a million dollars now has ultimately to do with the fact that – you guessed it – *inference rules don’t apply to statement components!* If inference rules did apply to statement components, we could easily constructs arguments like this, which would make our (rational) world a mess.

As an exercise, you might try to produce an argument to show why Example 1 is invalid.

Ex. Components: Please, carefully inspect the following derivations and mark those that are not permitted, justify those steps that are permitted.

1.	C → D	Pr.
2.	C • A	Pr.
3.	D	

1.	A → D	Pr.
2.	A → B	Pr.
3.	B	

1.	~C	Pr.
2.	~C → D	Pr.
3.	D	

1.	A → B	Pr.
2.	C • A	Pr.
3.	A	
4.	B	

1.	B → C	Pr.
2.	A • (B • C)	Pr.
3.	A	
4.	B	

1.	A → D	Pr.
2.	A → B	Pr.
3.	B	
4.	D	

1.	(A • B) • C	Pr.
2.	C → D	Pr.
3.	A	
4.	B	

1.	(B → C) • D	Pr.
2.	B	Pr.
3.	C	

1.	A → D	Pr.
2.	A	

1.	(A • B) ∨ C	Pr.
2.	C → D	Pr.
3.	A	
4.	B	

1.	(A • C) • C	Pr.
2.	C	

1.	~C • B	Pr.
2.	~C → D	Pr.
3.	~C	
4.	B	

5. Complex Instances of Modus Ponens, Simplification and Conjunction

We have already seen complex instances of •Int. and •Elim. We will look more carefully at the complex instance of →Elim in particular.

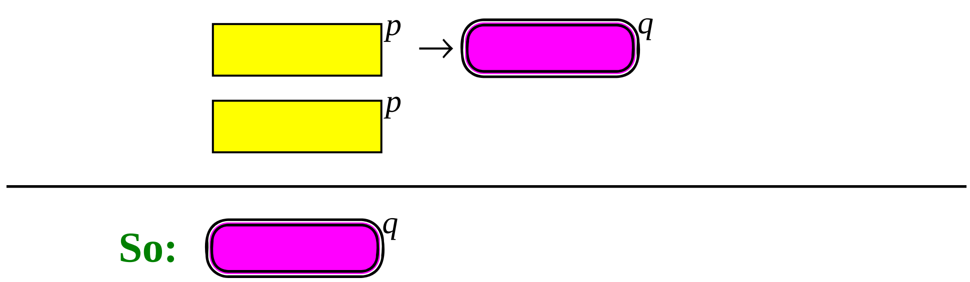
5.1. Complex Instances of Modus Ponens (→Elim)

Line *i*.	$p \rightarrow q$	
Line *j*.	p	
∴	q	**→Elim i, j**

The rule tells you: if you have two statement forms: a conditional ($p \rightarrow q$) and its antecedent (p), you are allowed to infer the consequent (q).

Instances of →Elim

Modus ponens, as all inference rules, is formulated in terms of statement variables. This means that it applies no matter how complex the statements are provided that they are instances of the →Elim argument form. Again, it might be useful to think of variables in terms of the boxes metaphor:



Here are some examples (in the right-hand column, statements in yellow are substitutions for variable *p*, statements in pink are substitutions for variable *q*):

Everywhere I talk about colors, do actually use your coloring pencils to mark them. Just remember to apply them correctly so that the variables match. You will thank yourself for indulging in this kindergarten task.

	$A \to B$			$A \to B$
	A			A
So	B		**So**	B

	$\sim A \to B$			$\sim A \to B$
	$\sim A$			$\sim A$
So	B		**So**	B

	$(A \lor B) \to (C \bullet A)$			$(A \lor B) \to (C \bullet A)$
	$A \lor B$			$A \lor B$
So	$C \bullet A$		**So**	$C \bullet A$

	$\sim[A \bullet (A \equiv B)] \to \sim\sim\sim C$			$\sim[A \bullet (A \equiv B)] \to \sim\sim\sim C$
	$\sim[A \bullet (A \equiv B)]$			$\sim[A \bullet (A \equiv B)]$
So	$\sim\sim\sim C$		**So**	$\sim\sim\sim C$

You should mark the inference patterns here.

Modus Ponens SAP

The "Modus Ponens" SAP (short animated presentation) is available as a PowerPoint show on-line. It should help you visualize also how to think about the more complex instances of →Elim.

Exercises on →Elim

Check your answers with *Solutions*. DO NOT postpone doing these exercises. Do them now!

Ex. →Elim-2.I. Please, fill in the missing information:

1.		Pr.
2.	C ∨ A	Pr.
3.	B	→Elim 1, 2

1.	A → (D • B)	Pr.
2.	A	Pr.
3.		→Elim 1, 2

1.	M → ~~N	Pr.
2.		Pr.
3.	~~N	→Elim 1, 2

1.	~~C → D	Pr.
2.		Pr.
3.		→Elim 1, 2

1.		Pr.
2.	~C → (A • B)	Pr.
3.		→Elim 1, 2

1.		Pr.
2.	A → B	Pr.
3.	C → D	→Elim 1, 2

1.	~D	Pr.
2.	(~D → A) • C	Pr.
3.	~D → A	Pr.
4.	A	

1.	~A → ~C	Pr.
2.	~A → D	Pr.
3.		Pr.
4.		→Elim 1, 3

1.	A → (A → B)	Pr.
2.	A	Pr.
3.		Pr.
4.	C	→Elim 1, 3

1.		Pr.
2.	A ∨ C	Pr.
3.	A	Pr.
4.	B	→Elim 1, 3

1.	~D → ~C	Pr.
2.	A ≡ C	Pr.
3.		Pr.
4.		→Elim 1, 3

1.	~(D • A)	Pr.
2.	(~D → A) → C	Pr.
3.		Pr.
4.	~C	→Elim 1,3

1.	A → B	Pr.
2.	B → C	Pr.
3.		Pr.
4.		→Elim 1, 3

1.		Pr.
2.	~(B ∨ C)	Pr.
3.	A → B	Pr.
4.	C	→Elim 1, 3

1.	~A → ~C	Pr.
2.	A → (D → (A → C))	Pr.
3.	A	Pr.
4.		→Elim 2,3

5.2. Examples of Proofs

Example 1

1. B → (A → D) Pr. ∴ ~C
2. (A → D) → ~C Pr.
3. B • D Pr.

Don't cry! Before you think about strategy, think about patterns. Can you see the conclusion somewhere in the premises? Yes, it is the consequent of the conditional in line 2. Mark it with it with red:

1. B → (A → D) Pr. ∴ ~C
2. (A → D) → ~C Pr.
3. B • D Pr.

The conclusion you need to infer on its own line is the consequent of the conditional in premise 2. The rule that allows you to infer a consequent of a conditional is →Elim, but you need the antecedent of that conditional on its own line. The antecedent of the conditional in line 2 is A → C. Mark it with blue – and also mark the other place where it occurs:

1. B → (A → D) Pr. ∴ ~C
2. (A → D) → ~C Pr.
3. B • D Pr.

You now see that it occurs in the consequent of a conditional in line 1. Again, you could apply →Elim to get it out on its own line, if you had the antecedent of the conditional in line 1, viz. B on its own line (you can mark its occurrences with green). You don't have it, but you can get it:

4. B •Elim. 3
5. A → D →Elim 1, 4
6. ~C →Elim 2, 5

That's it. The proof was not difficult – the difficult part was your seeing the relevant patterns. This is why it is important to use the colors to mark the same statements. Try to do the problem on your own – without peeking above – you are to prove ~C from the following premises:

1.	B → (A → D)	Pr.
2.	(A → D) → ~C	Pr.
3.	B • D	Pr.

Try a similar problem – here you are to prove ~(A → E) from the following premises:

1.	(A ∨ B) → [A ≡ (D • B)]	Pr.
2.	[A ≡ (D • B)] → ~(A → E)	Pr.
3.	(A ∨ B) • D	Pr.

Can you see that your proof would follow exactly in the same way? Read the above strategy and mark the patterns.

Example 2

Prove that A → D from the following premises:

1.	~B → [~C → (~B → (A → D))]	Pr.
2.	~B • ~C	Pr.

Here again it is useful to mark the place where A → D occurs. It occurs within the consequent of a consequent of a consequent of the conditional in line 1. The only way to get it out is by subsequent application of →Elim, but in order to start these applications you need to have the antecedent of the conditional in line 1, which is ~B. This you don't have. But if you look carefully at premise 2, you will see that this is a conjunction one of whose conjuncts is precisely ~B. So, let's start by deriving ~B, and using it to get the consequent out of line 1:

3. ~B	•Elim. 2
4. ~C → (~B → (A → D))	→Elim 1, 3

Now we are closer to getting A → D – it is in the consequent of a consequent of the conditional in line 4. Again we must apply →Elim to get the consequent (viz. ~B → (A → D)) of the conditional in line 4 on its own line. But to apply →Elim we need not only the conditional, which we have, but also its antecedent, which in the case of the conditional in line 4 is ~C. We don't have ~C. But we can get it by •Elim. from line 2.

5. ~C	•Elim. 2
6. ~B → (A → D)	→Elim 4, 5

We closer to the goal, but not yet there. The wanted statement A → D is now just one step away – it is the consequent of the conditional in line 6. To derive A → D we only need to have the antecedent of the conditional in line 6, viz. ~B. Do we have it? Yes, we do – in line 3. So the remaining step is:

7. A → D	→Elim 6, 3

Example 3

Prove that $(C \vee D) \equiv (C \bullet D)$ from the following premises:

1.	$[B \equiv (B \vee A)] \bullet [B \bullet (B \vee A)]$	Pr.
2.	$(B \vee A) \rightarrow [(C \vee D) \equiv (C \bullet D)]$	Pr.

Before you think about strategy, think about patterns. Can you see the conclusion somewhere in the premises? Yes, it is the consequent of the conditional in line 2. Mark it with it with red:

1. $[B \equiv (B \vee A)] \bullet [B \bullet (B \vee A)]$ Pr. $\therefore$ $(C \vee D) \equiv (C \bullet D)$
2. $(B \vee A) \rightarrow [(C \vee D) \equiv (C \bullet D)]$ Pr.

If you need to derive the consequent of a conditional, this most likely means that you will apply →Elim provided that you have the antecedent of the conditional, here $B \vee A$. Can you see $B \vee A$ somewhere else besides in line 1? Mark the occurrences with blue:

1. $[B \equiv (B \vee A)] \bullet [B \bullet (B \vee A)]$ Pr. $\therefore$ $(C \vee D) \equiv (C \bullet D)$
2. $(B \vee A) \rightarrow [(C \vee D) \equiv (C \bullet D)]$ Pr.

It occurs in two places in the first premise – as the second term of a biconditional, which is the first conjunct of premise 1, and as the second conjunct of the conjunction, which is itself the second conjunct of premise 1. If you think about it a little, you will see that you should apply **•Elim** to the first premise and infer the second conjunct:

3. $B \bullet (B \vee A)$ •Elim. 1

[Note that you could infer the first conjunct as well (i.e. derive $B \equiv (B \vee A)$ using •Elim. 1) but doing so would be pointless since we don't know any rules that would allow us to derive a term of a biconditional.] And now, we can obtain the second conjunct:

4. $B \vee A$ •Elim. 3

which is the antecedent of the conditional in line 2, hence:

5. $(C \vee D) \equiv (C \bullet D)$ →Elim 2, 4

Example 4

Prove that the following reasoning is valid:

> Amy and Susan both hate school. Whereas Amy is strong-willed – she will not go to school if she hates it; Susan is not – she will keep going to school even if she hates it. So, while Amy will not go to school, Susan will.

This is a piece of reasoning here, which we follow without much problems. And we can capture its validity using deductive reasoning, which involves just the three inference rules we have introduced. We need to symbolize the reasoning, let me use the following Key:

A – Amy hates school
S – Susan hates school
M – Amy will go to school
U – Susan will go to school

A • S
(A → ~M) • (S → U)
∴ ~M • U

Prepare the derivation:

1.	A • S	Pr.	∴ ~M • U
2.	(A → ~M) • (S → U)	Pr.	

Think about your strategy. You are to obtain a conjunction of ~M and U. You could obtain it if you had both these statements on their own, which you don't. Can you get them? Think about U first. U occurs as a consequent of a conditional, which is hidden from view in the conjunction in line 2. But you can apply •Elim to get the conditional (S → U) on its own and you get apply •Elim to get S on its own, which will then allow you to apply →Elim Thus:

3. S → U	•Elim. 2
4. S	•Elim. 1
5. U	→Elim 3, 4

An analogical strategy will allow you to obtain ~M on its own:

6. A → ~M	•Elim. 2
7. A	•Elim. 1
8. ~M	→Elim 6, 7

From there, it's only a matter of conjoining ~M and U

9. ~M • U	•Int. 8, 5

5.3. Exercises

Check your answers with *Solutions*.

Ex. →**Elim.•Int.•Elim.2.II.** The following proofs are missing exactly one step to prove the conclusion (on the last line). Please, fill in the missing step, justify it and justify the last step:

1.	(C • A) → D	Pr.
2.	D • (C • A)	Pr.
3.		
4.	D	

1.	(A • B) → C	Pr.
2.	A	Pr.
3.	B	Pr.
4.		
5.	C	

1.	(A → B) → C	Pr.
2.	(A → B) • D	Pr.
3.		
4.	C	

1.	A → [(B → C) → D]	Pr.
2.	A	Pr.
3.	B → C	Pr.
4.		
5.	D	

1.	(A → B) → (A → C)	Pr.
2.	A → B	Pr.
3.	A	Pr.
4.		
5.	C	

Ex. →**Elim.•Int.•Elim.2.III.** The following proofs are missing exactly two steps to prove the conclusion (on the last line). Please, fill in the missing steps, justify them and justify the last step:

1.	(B • D) → (D → C)	Pr.
2.	A • (B • D)	Pr.
3.	D	Pr.
4.		
5.		
6.	C	

1.	(~A • ~B) → (~B → C)	Pr.
2.	~B	Pr.
3.	~A	Pr.
4.		
5.		
6.	C	

1.	A → [A → (A → C)]	Pr.
2.	A	Pr.
3.		
4.		
5.	C	

1.	(A • B) → (C • D)	Pr.
2.	(A • B) • G	Pr.
3.		
4.		
5.	D	

1.	(K → L) → (K → M)	Pr.
2.	K → L	Pr.
3.	K	Pr.
4.		
5.		
6.	K • M	

1.	K → [(L → M) → N]	Pr.
2.	K	Pr.
3.	L → M	Pr.
4.		
5.		
6.	(L → M) • N	

Ex. →Elim.•Int.•Elim.2.IV. Prove that the indicated conclusion follows from the premises given:

Prove that C from:

1.	A → (B → C)	Pr.
2.	A → B	Pr.
3.	A	Pr.

Prove that A from:

1.	(~C → B) → (B → A)	Pr.
2.	~C	Pr.
3.	~C → B	Pr.

Prove that C from:

1.	B → (B → C)	Pr.
2.	A → (B • D)	Pr.
3.	A	Pr.

Prove that ~D from:

1.	(A • C) → (B ∨ C)	Pr.
2.	(B ∨ C) → ~D	Pr.
3.	(A • B) • C	Pr.

Prove that D from:

1.	(B • A) → D	Pr.
2.	(A • C) • B	Pr.

Prove that [(A →B) • (B → C)] • (A → B) from:

1.	A → B	Pr.
2.	B → C	Pr.

Prove that A from:

1.	B → [B → (B → A)]	Pr.
2.	B	Pr.

Prove that D from:

1.	(B ∨ C) → (A • D)	Pr.
2.	(B ∨ C) • A	Pr.

Prove that $(C \bullet D) \bullet (A \vee B)$ from:

1.	$(A \vee B) \to C$	Pr.
2.	$D \to (A \vee B)$	Pr.
3.	D	Pr.

Prove that $B \bullet C$ from:

1.	$A \to (A \to B)$	Pr.
2.	$(A \to B) \to C$	Pr.
3.	A	Pr.

Prove that $(A \bullet B) \bullet C$ from:

1.	$(A \vee B) \to C$	Pr.
2.	$A \to (A \to B)$	Pr.
3.	$(A \vee B) \bullet A$	Pr.

Prove that $M \bullet N$ from:

1.	$(A \bullet B) \to [(L \to M) \to N]$	Pr.
2.	$A \bullet (L \to M)$	Pr.
3.	$L \bullet B$	Pr.

FAQ: "May I add a premise?"

No! The premises in a proof correspond to the information that is given in the proof. This is the basis from which we can draw further information by means of inference rules until we reach the conclusion. You are never allowed to simply add a premise to a proof.

What You Need to Know and Do

- You need to know the inference rules and be able to apply them
- You need to be able to construct proofs using the inference rules