Solutions to Workbook Exercises

Unit 7:

Validity

Exercise "Proper Argument Form" – 1

Provide the proper argument forms for the following arguments

(a)
$$\begin{array}{l} A \rightarrow B \\ A \\ \hline B \end{array} \qquad \begin{array}{l} p \rightarrow q \\ p \\ \hline q \end{array}$$

(b)
$$\begin{array}{l} A \rightarrow B \\ B \\ \hline A \end{array} \qquad \begin{array}{l} p \rightarrow q \\ q \\ \hline p \end{array}$$

(c)
$$\begin{array}{l} A \rightarrow B \\ {\sim}A \\ \hline {\sim}B \end{array} \qquad \begin{array}{l} p \rightarrow q \\ {\sim}p \\ \hline {\sim}q \end{array}$$

(d)
$$\begin{array}{l} A \rightarrow B \\ {\sim}B \\ \hline {\sim}A \end{array} \qquad \begin{array}{l} p \rightarrow q \\ {\sim}q \\ \hline {\sim}p \end{array}$$

(e)
$$\begin{array}{l} A \rightarrow B \\ C \rightarrow {\sim}B \\ C \\ \hline {\sim}A \end{array} \qquad \begin{array}{l} p \rightarrow q \\ r \rightarrow {\sim}q \\ r \\ \hline {\sim}p \end{array}$$

(f)
$$\begin{array}{l} {\sim}A \bullet {\sim}B \\ A \vee C \\ D \vee B \\ \hline D \end{array} \qquad \begin{array}{l} {\sim}p \bullet {\sim}q \\ p \vee r \\ s \vee q \\ \hline s \end{array}$$

(g)
$$\begin{array}{l} (A \rightarrow B) \bullet (B \rightarrow A) \\ \hline A \equiv B \end{array} \qquad \begin{array}{l} (p \rightarrow q) \bullet (q \rightarrow p) \\ \hline p \equiv q \end{array}$$

(h)
$$\begin{array}{l} A \rightarrow B \\ B \rightarrow C \\ C \rightarrow A \\ \hline A \equiv C \end{array} \qquad \begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ r \rightarrow p \\ \hline p \equiv r \end{array}$$

(i)
$$\begin{array}{l} (A \equiv B) \bullet {\sim}B \\ (C \equiv D) \bullet {\sim}D \\ \hline {\sim}A \bullet {\sim}C \end{array} \qquad \begin{array}{l} (p \equiv q) \bullet {\sim}q \\ (r \equiv s) \bullet {\sim}s \\ \hline {\sim}p \bullet {\sim}r \end{array}$$

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Exercise “Proper Argument Form” – 2

Connect the arguments (expressed in English) with their proper argument forms.

B: Ann needs Bert’s help H: Bert helps Ann	L: Ann needs Bert’s love M: Ann will get married	P: Ann passes logic R: Ann will retake logic	S: Ann is very scared Y: Ann finds a boyfriend
$P \rightarrow (M \vee Y)$ $P \bullet {\sim}M$ ——— Y	$P \rightarrow {\sim}(B \bullet L)$ $P \bullet L$ ——— ${\sim}B$	$({\sim}P \rightarrow R) \bullet (P \rightarrow {\sim}R)$ ——— $R \equiv {\sim}P$	$({\sim}P \vee S) \rightarrow H$ ${\sim}H$ ——— $P \bullet {\sim}S$
If Ann passes logic then she will either get married or she will find herself another boyfriend. Ann passed logic but did not get married. So, Ann found herself another boyfriend.	If Ann passes logic then she will not need both Bert’s help and his love. Ann passed logic but she still needs Bert’s love. So, she does not need his help.	If Ann does not pass logic, then she will need to retake it, but if she passes then she will not need to retake it. So, Ann will need to retake logic if and only if she does not pass it.	If Ann does not pass logic the first time or is very scared, then Bert will do all he can to help her. Bert did not need to help Ann. So, Ann passed logic and was not very scared.
$({\sim}p \vee q) \rightarrow r$ ${\sim}r$ ——— $p \bullet {\sim}q$	$p \rightarrow (q \vee r)$ $p \bullet {\sim}q$ ——— r	$p \rightarrow {\sim}(q \bullet r)$ $p \bullet r$ ——— ${\sim}q$	$({\sim}p \rightarrow q) \bullet (p \rightarrow {\sim}q)$ ——— $q \equiv {\sim}p$
If Daryl does not water her plants, they will wilt, but if she does water them they will not wilt. So, the plants will wilt if and only if Daryl does not water them.	If Ben will no longer fight with his sister or if he will improve at maths, he will get a hamster. Ben did not get a hamster. So, he kept fighting with his sister and did not improve at maths.	If Chris buys ice-cream then he will eat it either immediately or within an hour. He bought the ice-cream but did not have the time to eat it immediately. So, he ate it within an hour.	If *X* is an integer then it is not both odd and even. *X* is an integer and it is even. So *X* is not odd.
$({\sim}W \rightarrow P) \bullet (W \rightarrow {\sim}P)$ ——— $P \equiv {\sim}W$	$({\sim}F \vee M) \rightarrow H$ ${\sim}H$ ——— $F \bullet {\sim}M$	$C \rightarrow (I \vee H)$ $C \bullet {\sim}I$ ——— H	$I \rightarrow {\sim}(O \bullet E)$ $I \bullet E$ ——— ${\sim}O$
P: Plants wilt W: Daryl waters her plants	F: Ben fights with his sister H: Ben gets hamster M: Ben improves at maths	C: Chris buys ice-cream H: Chris eats ice-cream in an hour I: Chris eats ice-cream immediately	E: *X* is even I: *X* is an integer O: *X* is odd

Optional Exercise "Counterexamples"

Show that each of the following arguments is invalid by producing a counterexample argument that shares the logical form of the original argument, but where all the premises are evidently true while the conclusion is evidently false.
(Note that it may be very hard for you to come up with good examples. If it is, skip the exercise for now.)

(a) If Susan gets 92 points on her quizzes she will get an A in logic.
Susan got an A in logic.
____
So, Susan got 92 points on her quizzes.

Counterexample:		Invalid argument form
If Dr. P. is a monkey then she is a mammal.	(True)	$p \to q$
Dr. P. is a mammal.	(True)	q
So, Dr. P. is a monkey.	(False)	p

(b) If Susan gets 92 points on her quizzes she will get an A in logic.
Susan did not get 92 points on her quizzes.
____
So, Susan did not get an A in logic

Counterexample:		Invalid argument form
If Dr. P. is a monkey then she is a mammal.	(True)	$p \to q$
Dr. P. is not a monkey.	(True)	$\sim p$
So, Dr. P. is not a mammal.	(False)	$\sim q$

(c) If government spending increases then the economy will crash.
If unemployment rises then the economy will crash
____
So, if government spending increases then unemployment will rise.

Counterexample:		Invalid argument form
If Dr. P. is a monkey then she is a mammal.	(True)	$p \to q$
If Dr. P. is a donkey then she is a mammal.	(True)	$r \to q$
So, if Dr. P. is a monkey then she is a donkey.	(False)	$p \to r$

Exercise "Invalidity"

(a)-(b)-(c) Table 1 has been completed in accordance with instructions. All counterexamples have been checked: ☑.

(d) The argument form γ is invalid because there are instances of that argument form with true premises and a false conclusion.

(e) The argument

If Lassie is a dog then Lassie is a mammal
Lassie is a mammal
____
Lassie is a dog

is invalid because the proper logical form of this argument is invalid. The proper logical form of the above argument is the argument form γ, which has been shown to be invalid.

Table 1: A few substitution instances of the argument form γ.

Substitution Key	Premise: $p \to q$	Premise: q	Conclusion: p	
⋮	⋮	⋮	⋮	
p:=Marilyn Monroe had hair (was not bald) q:= MM had blond hair	If MM had blond hair then she had hair. ⊗ true ○ false	MM had hair ⊗ true ○ false	MM had blond hair. ⊗ true ○ false	☐ counterexample
p:=Danny de Vito has hair q:= DdV has blond hair	If DdV has blond hair then he has hair. ⊗ true ○ false	DdV has hair ⊗ true ○ false	DdV has blond hair. ○ true ⊗ false	☑ counterexample
⋮	⋮	⋮	⋮	
p:=JFK was assassinated q:=JFK is dead	If JFK was assassinated then he is dead ⊗ true ○ false	JFK is dead ⊗ true ○ false	JFK was assassinated ⊗ true ○ false	☐ counterexample
p:=Brad Pitt was assassinated q:=Brad Pitt is dead	If Brad Pitt was assassinated then he is dead ⊗ true ○ false	Brad Pitt is dead ○ true ⊗ false	Brad Pitt was assassinated ○ true ⊗ false	☐ counterexample
⋮	⋮	⋮	⋮	
p:=Lassie is a dog q:=Lassie is a mammal	If Lassie is a dog then Lassie is a mammal ⊗ true ○ false	Lassie is a mammal ⊗ true ○ false	Lassie is a dog ⊗ true ○ false	☐ counterexample
p:=Flipper is a dog q:=Flipper is a mammal	If Flipper is a dog then Flipper is a mammal ⊗ true ○ false	Flipper is a mammal ⊗ true ○ false	Flipper is a dog ○ true ⊗ false	☑ counterexample
⋮	⋮	⋮	⋮	
p:= Ronald Reagan was assassinated q:= RR is dead	If RR was assassinated then RR is dead ⊗ true ○ false	RR is dead ⊗ true ○ false	RR was assassinated ○ true ⊗ false	☑ counterexample
p:= It is cloudy q:= It rains	If it is cloudy then it rains ○ true ⊗ false	It rains. ○ true ⊗ false	It is cloudy. ⊗ true ○ false	☐ counterexample
⋮	⋮	⋮	⋮	

Exercise “Validity – Dry”

Decide whether the argument form corresponding to the truth table you are given is valid or not.

(a)

Row	Premise 1	Premise 2	Conclusion	Is it a counterexample row?
1	F	T	T	○ yes (counterexample) ⊗ no
2	F	F	T	○ yes (counterexample) ⊗ no
3	T	T	F	⊗ yes (counterexample) ○ no
4	T	F	T	○ yes (counterexample) ⊗ no

The argument form for which this is a truth table is:
☑ invalid because there is at least one counterexample row (viz. row 3).

(b)

Row	Premise 1	Premise 2	Conclusion	Is it a counterexample row?
1	F	T	F	○ yes (counterexample) ⊗ no
2	T	T	T	○ yes (counterexample) ⊗ no
3	T	F	F	○ yes (counterexample) ⊗ no
4	T	T	F	⊗ yes (counterexample) ○ no

The argument form for which this is a truth table is:
☑ invalid because there is at least one counterexample row, viz. row 4.

(c)

Row	Prem. 1	Prem. 2	Prem. 3	Conclusion	Is it a counterexample row?
1	F	F	F	F	○ yes (counterexample) ⊗ no
2	T	F	F	T	○ yes (counterexample) ⊗ no
3	F	T	T	F	○ yes (counterexample) ⊗ no
4	T	F	F	T	○ yes (counterexample) ⊗ no

The argument form for which this is a truth table is:
☑ valid because there are no counterexample rows where all the premises are true and the conclusion is false.

(d)

Row	Premise 1	Conclusion	Is it a counterexample row?
1	F	F	○ yes (counterexample) ⊗ no
2	F	T	○ yes (counterexample) ⊗ no
3	T	F	⊗ yes (counterexample) ○ no
4	T	T	○ yes (counterexample) ⊗ no

The argument form for which this is a truth table is:
☑ invalid because there is at least one counterexample row, viz. row 3.

Example 2

Skinner's and Watson's theory are not both true.
In fact, Watson's theory is false.

So, Skinner's theory is true.

Symbolization (which may be helpful in deciding what the logical form of an argument is):

S: Skinner's theory is true
W: Watson's theory is true

$$\frac{\begin{array}{c}{\sim}S \lor {\sim}W \\ {\sim}W\end{array}}{S} \qquad \text{or} \qquad \frac{\begin{array}{c}{\sim}(S \bullet W) \\ {\sim}W\end{array}}{S}$$

Logical form of the argument:

$$\frac{\begin{array}{c}{\sim}p \lor {\sim}q \\ {\sim}q\end{array}}{p} \qquad \text{or} \qquad \frac{\begin{array}{c}{\sim}(p \bullet q) \\ {\sim}q\end{array}}{p}$$

Calculation truth table:

	p	q	${\sim}p \lor {\sim}q$			${\sim}q$		p
1	T	T	~T ∨ ~T	F ∨ F	F	~T	F	T
2	T	F	~T ∨ ~F	F ∨ T	T	~F	T	T
3	F	T	~F ∨ ~T	T ∨ F	T	~T	F	F
4	F	F	~F ∨ ~F	T ∨ T	T	~F	T	F

Summary truth table:

	${\sim}p \lor {\sim}q$	${\sim}q$	p	**Is it a counterexample row?**
1	F	F	T	○ yes ⊗ no
2	T	T	T	○ yes ⊗ no
3	T	F	F	○ yes ⊗ no
4	T	T	F	⊗ yes ○ no

The argument form is invalid because there is at least one row (viz. row 4) where all the premises are true and the conclusion is false.

Exercise "Validity – 1"

Complete the truth tables to decide whether a given argument form is valid or not.

(a) $p \rightarrow q$
$\sim p$
$\overline{}$
$\sim q$

	p	q	$p \rightarrow q$		$\sim p$		$\sim q$	
1	T	T	T → T	T	~T	F	~T	F
2	T	F	T → F	F	~T	F	~F	T
3	F	T	F → T	T	~F	T	~T	F
4	F	F	F → F	T	~F	T	~F	T

Summary truth table:

	$p \rightarrow q$	$\sim p$	$\sim q$	**Is it a counterexample row?**
1	T	F	F	○ yes ⊗ no
2	F	F	T	○ yes ⊗ no
3	T	T	F	⊗ yes ○ no
4	T	T	T	○ yes ⊗ no

The argument form is invalid because there is at least one counterexample row (row 3).

(b) $p \equiv p$
$\overline{}$
p

	p	$p \equiv p$		p
1	T	T ≡ T	T	T
2	F	F ≡ F	T	F

Summary truth table:

	$p \equiv p$	p	**Is it a counterexample row?**
1	T	T	○ yes ⊗ no
2	T	F	⊗ yes ○ no

The argument form is invalid because there is at least one counterexample row (row 2).

(c) $p \rightarrow (q \rightarrow r)$

$p \rightarrow \sim q$

$\sim r$

	p	q	r	$p \rightarrow (q \rightarrow r)$			$p \rightarrow \sim q$			$\sim r$	
1	T	T	T	T → (T → T)	T → (T)	T	T → ~T	T → F	F	~T	F
2	T	T	F	T → (T → F)	T → (F)	F	T → ~T	T → F	F	~F	T
3	T	F	T	T → (F → T)	T → (T)	T	T → ~F	T → T	T	~T	F
4	T	F	F	T → (F → F)	T → (T)	T	T → ~F	T → T	T	~F	T
5	F	T	T	F → (T → T)	F → (T)	T	F → ~T	F → F	T	~T	F
6	F	T	F	F → (T → F)	F → (F)	T	F → ~T	F → F	T	~F	T
7	F	F	T	F → (F → T)	F → (T)	T	F → ~F	F → T	T	~T	F
8	F	F	F	F → (F → F)	F → (T)	T	F → ~F	F → T	T	~F	T

Summary truth table:

	$p \rightarrow (q \rightarrow r)$	$p \rightarrow \sim q$	$\sim r$	**Is it a counterexample row?**
1	T	F	F	○ yes ⊗ no
2	F	F	T	○ yes ⊗ no
3	T	T	F	⊗ yes ○ no
4	T	T	T	○ yes ⊗ no
5	T	T	F	⊗ yes ○ no
6	T	T	T	○ yes ⊗ no
7	T	T	F	⊗ yes ○ no
8	T	T	T	○ yes ⊗ no

The argument form is invalid because there is at least one counterexample row (in fact there are three counterexample rows 3, 5 and 7).

(d) $p \equiv p$

$p \rightarrow p$

	p	$p \equiv p$		$p \rightarrow p$	
1	T	T ≡ T	T	T → T	T
2	F	F ≡ F	T	F → F	T

Summary truth table:

	$p \equiv p$	$p \rightarrow p$	**Is it a counterexample row?**
1	T	T	○ yes ⊗ no
2	T	T	○ yes ⊗ no

The argument form is valid because there are no counterexample rows where all the premises (here: just one) are true while the conclusion is false.

(e) $p \vee q$
$\sim q$
———
p

	p	q	$p \vee q$		$\sim q$		p
1	T	T	T ∨ T	T	~T	F	T
2	T	F	T ∨ F	T	~F	T	T
3	F	T	F ∨ T	T	~T	F	F
4	F	F	F ∨ F	F	~F	T	F

Summary truth table:

	$p \vee q$	$\sim q$	p	**Is it a counterexample row?**
1	T	F	T	○ yes ⊗ no
2	T	T	T	○ yes ⊗ no
3	T	F	F	○ yes ⊗ no
4	F	T	F	○ yes ⊗ no

The argument form is valid because there are no counterexample rows where all the premises (here: just one) are true while the conclusion is false.

(f) $p \rightarrow q$
$\sim q$
———
$\sim p$

	p	q	$p \rightarrow q$		$\sim q$		$\sim p$	
1	T	T	T → T	T	~T	F	~T	F
2	T	F	T → F	F	~F	T	~T	F
3	F	T	F → T	T	~T	F	~F	T
4	F	F	F → F	T	~F	T	~F	T

Summary truth table:

	$p \rightarrow q$	$\sim q$	$\sim p$	**Is it a counterexample row?**
1	T	F	F	○ yes ⊗ no
2	F	T	F	○ yes ⊗ no
3	T	F	T	○ yes ⊗ no
4	T	T	T	○ yes ⊗ no

The argument form is valid because there are no counterexample rows where all the premises (here: just one) are true while the conclusion is false.

Exercise "Validity – 2"

(a) $\dfrac{p \to q}{\sim p \to \sim q}$

	p	q	$p \to q$		$\sim p \to \sim q$		
1	T	T	T → T	T	~T → ~T	F → F	T
2	T	F	T → F	F	~T → ~F	F → T	T
3	F	T	F → T	T	~F → ~T	T → F	F
4	F	F	F → F	T	~F → ~F	T → T	T

Summary truth table:

	$p \to q$	$\sim p \to \sim q$	**Is it a counterexample row?**
1	T	T	○ yes ⊗ no
2	F	T	○ yes ⊗ no
3	T	F	⊗ yes ○ no
4	T	T	○ yes ⊗ no

The argument form is invalid because there are is at least one counterexample row (row 3).

(b) $\dfrac{p \to q}{\sim q \to \sim p}$

	p	q	$p \to q$		$\sim q \to \sim p$		
1	T	T	T → T	T	~T → ~T	F → F	T
2	T	F	T → F	F	~F → ~T	T → F	F
3	F	T	F → T	T	~T → ~F	F → T	T
4	F	F	F → F	T	~F → ~F	T → T	T

Summary truth table:

	$p \to q$	$\sim q \to \sim p$	**Is it a counterexample row?**
1	T	T	○ yes ⊗ no
2	F	F	○ yes ⊗ no
3	T	T	○ yes ⊗ no
4	T	T	○ yes ⊗ no

The argument form is valid because there are no counterexample rows.

(c) $\dfrac{p}{p \bullet q}$

	p	q	p	$p \bullet q$	
1	T	T	T	T • T	T
2	T	F	T	T • F	F
3	F	T	F	F • T	F
4	F	F	F	F • F	F

Summary truth table:

	p	$p \bullet q$	**Is it a counterexample row?**
1	T	T	○ yes ⊗ no
2	T	F	⊗ yes ○ no
3	F	F	○ yes ⊗ no
4	F	F	○ yes ⊗ no

The argument form is invalid because there are is at least one counterexample row (row 2).

(d) $\dfrac{p}{p \vee q}$

	p	q	p	$p \vee q$	
1	T	T	T	T ∨ T	T
2	T	F	T	T ∨ F	T
3	F	T	F	F ∨ T	T
4	F	F	F	F ∨ F	F

Summary truth table:

	p	$p \vee q$	**Is it a counterexample row?**
1	T	T	○ yes ⊗ no
2	T	T	○ yes ⊗ no
3	F	T	○ yes ⊗ no
4	F	F	○ yes ⊗ no

The argument form is valid because there are no counterexample rows.

(e) $\dfrac{p}{p \rightarrow q}$

	p	q	p	$p \rightarrow q$	
1	T	T	T	T → T	T
2	T	F	T	T → F	F
3	F	T	F	F → T	T
4	F	F	F	F → F	T

Summary truth table:

	p	$p \rightarrow q$	**Is it a counterexample row?**
1	T	T	○ yes ⊗ no
2	T	F	⊗ yes ○ no
3	F	T	○ yes ⊗ no
4	F	T	○ yes ⊗ no

The argument form is invalid because there are is at least one counterexample row (row 2).

(f) $\dfrac{p}{q \rightarrow p}$

	p	q	p	$q \rightarrow p$	
1	T	T	T	T → T	T
2	T	F	T	F → T	T
3	F	T	F	T → F	F
4	F	F	F	F → F	T

Summary truth table:

	p	$q \rightarrow p$	**Is it a counterexample row?**
1	T	T	○ yes ⊗ no
2	T	T	○ yes ⊗ no
3	F	F	○ yes ⊗ no
4	F	T	○ yes ⊗ no

The argument form is valid because there are no counterexample rows.

(g) $p \to q$
$q \to r$
$\overline{}$
$p \to r$

	p	q	r	$p \to q$		$q \to r$		$p \to r$	
1	T	T	T	T → T	T	T → T	T	T → T	T
2	T	T	F	T → T	T	T → F	F	T → F	F
3	T	F	T	T → F	F	F → T	T	T → T	T
4	T	F	F	T → F	F	F → F	T	T → F	F
5	F	T	T	F → T	T	T → T	T	F → T	T
6	F	T	F	F → T	T	T → F	F	F → F	T
7	F	F	T	F → F	T	F → T	T	F → T	T
8	F	F	F	F → F	T	F → F	T	F → F	T

Summary truth table:

	$p \to q$	$q \to r$	$p \to r$	**Is it a counterexample row?**
1	T	T	T	○ yes ⊗ no
2	T	F	F	○ yes ⊗ no
3	F	T	T	○ yes ⊗ no
4	F	T	F	○ yes ⊗ no
5	T	T	T	○ yes ⊗ no
6	T	F	T	○ yes ⊗ no
7	T	T	T	○ yes ⊗ no
8	T	T	T	○ yes ⊗ no

The argument form is valid because there are no counterexample rows where both premises are true while the conclusion is false.

(h) $p \to q$
$p \to r$
$\overline{}$
$q \to r$

	p	q	r	$p \to q$		$p \to r$		$q \to r$	
1	T	T	T	T → T	T	T → T	T	T → T	T
2	T	T	F	T → T	T	T → F	F	T → F	F
3	T	F	T	T → F	F	T → T	T	F → T	T
4	T	F	F	T → F	F	T → F	F	F → F	T
5	F	T	T	F → T	T	F → T	T	T → T	T
6	F	T	F	F → T	T	F → F	T	T → F	F
7	F	F	T	F → F	T	F → T	T	F → T	T
8	F	F	F	F → F	T	F → F	T	F → F	T

Summary truth table:

	$p \to q$	$p \to r$	$q \to r$	**Is it a counterexample row?**
1	T	T	T	○ yes ⊗ no
2	T	F	F	○ yes ⊗ no
3	F	T	T	○ yes ⊗ no
4	F	F	T	○ yes ⊗ no
5	T	T	T	○ yes ⊗ no
6	T	T	F	⊗ yes ○ no
7	T	T	T	○ yes ⊗ no
8	T	T	T	○ yes ⊗ no

The argument form is invalid because there is at least one counterexample row (row 6).

(i) $p \to r$

$q \to r$

$(p \vee q) \to r$

	p	q	r	$p \to r$		$q \to r$		$(p \vee q) \to r$		
1	T	T	T	T → T	T	T → T	T	(T ∨ T) → T	(T) → T	T
2	T	T	F	T → F	F	T → F	F	(T ∨ T) → F	(T) → F	F
3	T	F	T	T → T	T	F → T	T	(T ∨ F) → T	(T) → T	T
4	T	F	F	T → F	F	F → F	T	(T ∨ F) → F	(T) → F	F
5	F	T	T	F → T	T	T → T	T	(F ∨ T) → T	(T) → T	T
6	F	T	F	F → F	T	T → F	F	(F ∨ T) → F	(T) → F	F
7	F	F	T	F → T	T	F → T	T	(F ∨ F) → T	(F) → T	T
8	F	F	F	F → F	T	F → F	T	(F ∨ F) → F	(F) → F	T

Summary truth table:

	$p \to r$	$q \to r$	$(p \vee q) \to r$	**Is it a counterexample row?**
1	T	T	T	○ yes ⊗ no
2	F	F	F	○ yes ⊗ no
3	T	T	T	○ yes ⊗ no
4	F	T	F	○ yes ⊗ no
5	T	T	T	○ yes ⊗ no
6	T	F	F	○ yes ⊗ no
7	T	T	T	○ yes ⊗ no
8	T	T	T	○ yes ⊗ no

The argument form is valid because there is no counterexample row in its truth table.

(j) $p \to q$
$r \to s$
$p \vee r$
———
$q \vee s$

	p	q	r	s	$p \to q$		$r \to s$		$p \vee r$		$q \vee s$	
1	T	T	T	T	T → T	T	T → T	T	T ∨ T	T	T ∨ T	T
2	T	T	T	F	T → T	T	T → F	F	T ∨ T	T	T ∨ F	T
3	T	T	F	T	T → T	T	F → T	T	T ∨ F	T	T ∨ T	T
4	T	T	F	F	T → T	T	F → F	T	T ∨ F	T	T ∨ F	T
5	T	F	T	T	T → F	F	T → T	T	T ∨ T	T	F ∨ T	T
6	T	F	T	F	T → F	F	T → F	F	T ∨ T	T	F ∨ F	F
7	T	F	F	T	T → F	F	F → T	T	T ∨ F	T	F ∨ T	T
8	T	F	F	F	T → F	F	F → F	T	T ∨ F	T	F ∨ F	F
9	F	T	T	T	F → T	T	T → T	T	F ∨ T	T	T ∨ T	T
10	F	T	T	F	F → T	T	T → F	F	F ∨ T	T	T ∨ F	T
11	F	T	F	T	F → T	T	F → T	T	F ∨ F	F	T ∨ T	T
12	F	T	F	F	F → T	T	F → F	T	F ∨ F	F	T ∨ F	T
13	F	F	T	T	F → F	T	T → T	T	F ∨ T	T	F ∨ T	T
14	F	F	T	F	F → F	T	T → F	F	F ∨ T	T	F ∨ F	F
15	F	F	F	T	F → F	T	F → T	T	F ∨ F	F	F ∨ T	T
16	F	F	F	F	F → F	T	F → F	T	F ∨ F	F	F ∨ F	F

Summary truth table:

	$p \to q$	$r \to s$	$p \vee q$	$q \vee s$	**Is it a counterexample row?**
1	T	T	T	T	○ yes ⊗ no
2	T	F	T	T	○ yes ⊗ no
3	T	T	T	T	○ yes ⊗ no
4	T	T	T	T	○ yes ⊗ no
5	F	T	T	T	○ yes ⊗ no
6	F	F	T	F	○ yes ⊗ no
7	F	T	T	T	○ yes ⊗ no
8	F	T	T	F	○ yes ⊗ no
9	T	T	T	T	○ yes ⊗ no
10	T	F	T	T	○ yes ⊗ no
11	T	T	F	T	○ yes ⊗ no
12	T	T	F	T	○ yes ⊗ no
13	T	T	T	T	○ yes ⊗ no
14	T	F	T	F	○ yes ⊗ no
15	T	T	F	T	○ yes ⊗ no
16	T	T	F	F	○ yes ⊗ no

The argument form is valid because there is no counterexample row in its truth table.

(k) $p \to q$
$(p \bullet q) \to r$
$p \to (r \to s)$
$p \to s$

	p	q	r	s	$p \to q$		$(p \bullet q) \to r$			$p \to (r \to s)$			$p \to s$	
1	T	T	T	T	T → T	T	(T • T) → T	(T) → T	T	T → (T → T)	T → (T)	T	T → T	T
2	T	T	T	F	T → T	T	(T • T) → T	(T) → T	T	T → (T → F)	T → (F)	F	T → F	F
3	T	T	F	T	T → T	T	(T • T) → F	(T) → F	F	T → (F → T)	T → (T)	T	T → T	T
4	T	T	F	F	T → T	T	(T • T) → F	(T) → F	F	T → (F → F)	T → (T)	T	T → F	F
5	T	F	T	T	T → F	F	(T • F) → T	(F) → T	T	T → (T → T)	T → (T)	T	T → T	T
6	T	F	T	F	T → F	F	(T • F) → T	(F) → T	T	T → (T → F)	T → (F)	F	T → F	F
7	T	F	F	T	T → F	F	(T • F) → F	(F) → F	T	T → (F → T)	T → (T)	T	T → T	T
8	T	F	F	F	T → F	F	(T • F) → F	(F) → F	T	T → (F → F)	T → (T)	T	T → F	F
9	F	T	T	T	F → T	T	(F • T) → T	(F) → T	T	F → (T → T)	F → (T)	T	F → T	T
10	F	T	T	F	F → T	T	(F • T) → T	(F) → T	T	F → (T → F)	F → (F)	T	F → F	T
11	F	T	F	T	F → T	T	(F • T) → F	(F) → F	T	F → (F → T)	F → (T)	T	F → T	T
12	F	T	F	F	F → T	T	(F • T) → F	(F) → F	T	F → (F → F)	F → (T)	T	F → F	T
13	F	F	T	T	F → F	T	(F • F) → T	(F) → T	T	F → (T → T)	F → (T)	T	F → T	T
14	F	F	T	F	F → F	T	(F • F) → T	(F) → T	T	F → (T → F)	F → (F)	T	F → F	T
15	F	F	F	T	F → F	T	(F • F) → F	(F) → F	T	F → (F → T)	F → (T)	T	F → T	T
16	F	F	F	F	F → F	T	(F • F) → F	(F) → F	T	F → (F → F)	F → (T)	T	F → F	T

Summary truth table:

	$p \to q$	$(p \bullet q) \to r$	$p \to (r \to s)$	$p \to s$	**Is it a counterexample row?**
1	T	T	T	T	○ yes ⊗ no
2	T	T	F	F	○ yes ⊗ no
3	T	F	T	T	○ yes ⊗ no
4	T	F	T	F	○ yes ⊗ no
5	F	T	T	T	○ yes ⊗ no
6	F	T	F	F	○ yes ⊗ no
7	F	T	T	T	○ yes ⊗ no
8	F	T	T	F	○ yes ⊗ no
9	T	T	T	T	○ yes ⊗ no
10	T	T	T	T	○ yes ⊗ no
11	T	T	T	T	○ yes ⊗ no
12	T	T	T	T	○ yes ⊗ no
13	T	T	T	T	○ yes ⊗ no
14	T	T	T	T	○ yes ⊗ no
15	T	T	T	T	○ yes ⊗ no
16	T	T	T	T	○ yes ⊗ no

The argument form is valid because there is no counterexample row in its truth table.

(l) $p \to (q \to r)$
$q \to (p \to s)$
$p \to s$

	p	q	r	s	$p \to (q \to r)$			$q \to (p \to s)$			$p \to s$	
1	T	T	T	T	$T \to (T \to T)$	$T \to (T)$	T	$T \to (T \to T)$	$T \to (T)$	T	$T \to T$	T
2	T	T	T	F	$T \to (T \to T)$	$T \to (T)$	T	$T \to (T \to F)$	$T \to (F)$	F	$T \to F$	F
3	T	T	F	T	$T \to (T \to F)$	$T \to (F)$	F	$T \to (T \to T)$	$T \to (T)$	T	$T \to T$	T
4	T	T	F	F	$T \to (T \to F)$	$T \to (F)$	F	$T \to (T \to F)$	$T \to (F)$	F	$T \to F$	F
5	T	F	T	T	$T \to (F \to T)$	$T \to (T)$	T	$F \to (T \to T)$	$F \to (T)$	T	$T \to T$	T
6	T	F	T	F	$T \to (F \to T)$	$T \to (T)$	T	$F \to (T \to F)$	$F \to (F)$	T	$T \to F$	F
7	T	F	F	T	$T \to (F \to F)$	$T \to (T)$	T	$F \to (T \to T)$	$F \to (T)$	T	$T \to T$	T
8	T	F	F	F	$T \to (F \to F)$	$T \to (T)$	T	$F \to (T \to F)$	$F \to (F)$	T	$T \to F$	F
9	F	T	T	T	$F \to (T \to T)$	$F \to (T)$	T	$T \to (F \to T)$	$T \to (T)$	T	$F \to T$	T
10	F	T	T	F	$F \to (T \to T)$	$F \to (T)$	T	$T \to (F \to F)$	$T \to (T)$	T	$F \to F$	T
11	F	T	F	T	$F \to (T \to F)$	$F \to (F)$	T	$T \to (F \to T)$	$T \to (T)$	T	$F \to T$	T
12	F	T	F	F	$F \to (T \to F)$	$F \to (F)$	T	$T \to (F \to F)$	$T \to (T)$	T	$F \to F$	T
13	F	F	T	T	$F \to (F \to T)$	$F \to (T)$	T	$F \to (F \to T)$	$F \to (T)$	T	$F \to T$	T
14	F	F	T	F	$F \to (F \to T)$	$F \to (T)$	T	$F \to (F \to F)$	$F \to (T)$	T	$F \to F$	T
15	F	F	F	T	$F \to (F \to F)$	$F \to (T)$	T	$F \to (F \to T)$	$F \to (T)$	T	$F \to T$	T
16	F	F	F	F	$F \to (F \to F)$	$F \to (T)$	T	$F \to (F \to F)$	$F \to (T)$	T	$F \to F$	T

Summary truth table:

	$p \to (q \to r)$	$q \to (p \to s)$	$p \to s$	**Is it a counterexample row?**
1	T	T	T	○ yes ⊗ no
2	T	F	F	○ yes ⊗ no
3	F	T	T	○ yes ⊗ no
4	F	F	F	○ yes ⊗ no
5	T	T	T	○ yes ⊗ no
6	T	T	F	⊗ yes ○ no
7	T	T	T	○ yes ⊗ no
8	T	T	F	⊗ yes ○ no
9	T	T	T	○ yes ⊗ no
10	T	T	T	○ yes ⊗ no
11	T	T	T	○ yes ⊗ no
12	T	T	T	○ yes ⊗ no
13	T	T	T	○ yes ⊗ no
14	T	T	T	○ yes ⊗ no
15	T	T	T	○ yes ⊗ no
16	T	T	T	○ yes ⊗ no

The argument form is invalid because there is at least one counterexample row in its truth table (in fact there are two such rows, row 6 and 8).

Exercise "Validity – 3" – Symbolizations and Proper Logical Forms

Here are just the symbolizations and proper logical forms of the arguments in Ex. Validity – 3. Please check below for complete solutions.

(a)

① Symbolization:

A: Henry Kissinger advises the White House
R: Henry Kissinger is retired.
W: Bob Woodward has the right sources

$$\begin{array}{l} W \rightarrow (A \vee \sim R) \\ \underline{R \qquad\qquad} \\ \sim W \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} p \rightarrow (q \vee \sim r) \\ \underline{r \qquad\qquad} \\ \sim p \end{array}$$

(b)

① Symbolization:

B: Bulgaria will not join the EU.
R: Russia joins the EU
U: Rumania joins the EU

$$\begin{array}{l} R \rightarrow (U \rightarrow B) \\ \underline{U \qquad\qquad} \\ \sim B \rightarrow \sim R \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} p \rightarrow (q \rightarrow r) \\ \underline{q \qquad\qquad} \\ \sim r \rightarrow \sim p \end{array}$$

(c)

① Symbolization:

B: Bulgaria will not join the EU.
R: Russia joins the EU
U: Rumania joins the EU

$$\begin{array}{l} \sim R \rightarrow (U \rightarrow \sim B) \\ \underline{U \qquad\qquad} \\ B \rightarrow R \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} \sim p \rightarrow (q \rightarrow \sim r) \\ \underline{q \qquad\qquad} \\ r \rightarrow p \end{array}$$

(d)

① Symbolization:

B: USA builds the wall on the Mexican border
D: US-Mexico diplomatic relations will suffer.
I: Illegal immigration problem will be solved

$$\begin{array}{l} B \rightarrow I \\ \underline{B \rightarrow D} \\ I \rightarrow D \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} p \rightarrow q \\ \underline{p \rightarrow r} \\ q \rightarrow r \end{array}$$

(e)

① Symbolization:

B: USA builds the wall on the Mexican border
D: US-Mexico diplomatic relations will suffer.
I: Illegal immigration problem will be solved
O: Immigrants find other way to get to US

$$\begin{array}{l} B \rightarrow (\sim O \rightarrow I) \\ \underline{B \rightarrow D} \\ B \rightarrow (I \bullet D) \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} p \rightarrow (\sim q \rightarrow r) \\ \underline{p \rightarrow s} \\ p \rightarrow (r \bullet s) \end{array}$$

(f)

① Symbolization:

D: The universe is essentially deterministic.
P: All of a person's actions can be predicted.
R: People are entirely rational.

$$\begin{array}{l} D \rightarrow P \\ \underline{R \rightarrow P} \\ R \rightarrow \sim D \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} p \rightarrow q \\ \underline{r \rightarrow q} \\ r \rightarrow \sim p \end{array}$$

(g)

① Symbolization:

A: You should file tax form 12A this year
C: You should file tax form 12C this year
B: you filed tax form 40B last year.
O: Your income exceeded $40000.

$$\begin{array}{l} (A \vee C) \rightarrow B \\ \underline{(A \rightarrow O) \bullet (C \rightarrow \sim O)} \\ O \rightarrow A \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} (p \vee q) \rightarrow r \\ \underline{(p \rightarrow s) \bullet (q \rightarrow \sim s)} \\ s \rightarrow p \end{array}$$

Exercise "Validity – 3" – Complete Solutions

(a) If Bob Woodward has the right sources then Henry Kissinger either advises the White House or is not retired. But Henry Kissinger is retired. So, Bob Woodward does not have the right sources.

① Symbolization:

A: Henry Kissinger advises the White House
R: Henry Kissinger is retired.
W: Bob Woodward has the right sources

$$\begin{array}{l} W \to (A \vee \sim R) \\ \underline{R \qquad\qquad} \\ \sim W \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} p \to (q \vee \sim r) \\ \underline{r \qquad\qquad} \\ \sim p \end{array}$$

③	*p*	*q*	*r*	*p* → (*q* ∨ ~*r*)				*r*	~p	
1	T	T	T	T → (T ∨ ~T)	T → (T ∨ F)	T → (T)	T	T	~T	F
2	T	T	F	T → (T ∨ ~F)	T → (T ∨ T)	T → (T)	T	F	~T	F
3	T	F	T	T → (F ∨ ~T)	T → (F ∨ F)	T → (F)	F	T	~T	F
4	T	F	F	T → (F ∨ ~F)	T → (F ∨ T)	T → (T)	T	F	~T	F
5	F	T	T	F → (T ∨ ~T)	F → (T ∨ F)	F → (T)	T	T	~F	T
6	F	T	F	F → (T ∨ ~F)	F → (T ∨ T)	F → (T)	T	F	~F	T
7	F	F	T	F → (F ∨ ~T)	F → (F ∨ F)	F → (F)	T	T	~F	T
8	F	F	F	F → (F ∨ ~F)	F → (F ∨ T)	F → (T)	T	F	~F	T

Summary truth table:

	p → (*q* ∨ ~*r*)	*r*	~*p*	**Is it a counterexample row?**
1	T	T	F	⊗ yes ○ no
2	T	F	F	○ yes ⊗ no
3	F	T	F	○ yes ⊗ no
4	T	F	F	○ yes ⊗ no
5	T	T	T	○ yes ⊗ no
6	T	F	T	○ yes ⊗ no
7	T	T	T	○ yes ⊗ no
8	T	F	T	○ yes ⊗ no

④ The argument form is invalid ⑤ because there is at least one counterexample row in its truth table, viz. row 1.

⑥ The argument is thus invalid because its proper logical form is invalid.

(b) If Russia joins the European Union (the EU) then, if Rumania joins the EU then Bulgaria will join the EU. Rumania will join the EU. So if Bulgaria does not join the EU then Russia did not join the EU either.

① Symbolization:

B: Bulgaria will not join the EU.
R: Russia joins the EU
U: Rumania joins the EU

$$\begin{array}{l} R \to (U \to B) \\ \underline{U\qquad\qquad} \\ \sim B \to \sim R \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} p \to (q \to r) \\ \underline{q\qquad\qquad} \\ \sim r \to \sim p \end{array}$$

③	p	q	r	$p \to (q \to r)$			q	$\sim r \to \sim p$		
1	T	T	T	$T \to (T \to T)$	$T \to (T)$	T	T	$\sim T \to \sim T$	$F \to F$	T
2	T	T	F	$T \to (T \to F)$	$T \to (F)$	F	T	$\sim F \to \sim T$	$T \to F$	F
3	T	F	T	$T \to (F \to T)$	$T \to (T)$	T	F	$\sim T \to \sim T$	$F \to F$	T
4	T	F	F	$T \to (F \to F)$	$T \to (T)$	T	F	$\sim F \to \sim T$	$T \to F$	F
5	F	T	T	$F \to (T \to T)$	$F \to (T)$	T	T	$\sim T \to \sim F$	$F \to T$	T
6	F	T	F	$F \to (T \to F)$	$F \to (F)$	T	T	$\sim F \to \sim F$	$T \to T$	T
7	F	F	T	$F \to (F \to T)$	$F \to (T)$	T	F	$\sim T \to \sim F$	$F \to T$	T
8	F	F	F	$F \to (F \to F)$	$F \to (T)$	T	F	$\sim F \to \sim F$	$T \to T$	T

Summary truth table:

	$p \to (q \to r)$	q	$\sim r \to \sim p$	**Is it a counterexample row?**
1	T	T	T	○ yes ⊗ no
2	F	T	F	○ yes ⊗ no
3	T	F	T	○ yes ⊗ no
4	T	F	F	○ yes ⊗ no
5	T	T	T	○ yes ⊗ no
6	T	T	T	○ yes ⊗ no
7	T	F	T	○ yes ⊗ no
8	T	F	T	○ yes ⊗ no

④ The argument form is valid ⑤ because there are no counterexample rows in its truth table.

⑥ The argument is thus valid because its proper logical form is valid.

(c) If Russia does not join the European Union (the EU) then, if Rumania joins the EU then Bulgaria will not join the EU. Rumania will join the EU. So if Bulgaria joins the EU then Russia will join the EU as well.

① Symbolization:

B: Bulgaria will not join the EU.
R: Russia joins the EU
U: Rumania joins the EU

$$\begin{array}{l} \sim R \to (U \to \sim B) \\ \underline{U \qquad\qquad\qquad} \\ B \to R \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} \sim p \to (q \to \sim r) \\ \underline{q \qquad\qquad\qquad} \\ r \to p \end{array}$$

③	p	q	r	$\sim p \to (q \to \sim r)$				q	$r \to p$	
1	T	T	T	~T → (T → ~T)	F → (T → F)	F → (F)	T	T	T → T	T
2	T	T	F	~T → (T → ~F)	F → (T → T)	F → (T)	T	T	F → T	T
3	T	F	T	~T → (F → ~T)	F → (F → F)	F → (T)	T	F	T → T	T
4	T	F	F	~T → (F → ~F)	F → (F → T)	F → (T)	T	F	F → T	T
5	F	T	T	~F → (T → ~T)	T → (T → F)	T → (F)	F	T	T → F	F
6	F	T	F	~F → (T → ~F)	T → (T → T)	T → (T)	T	T	F → F	T
7	F	F	T	~F → (F → ~T)	T → (F → F)	T → (T)	T	F	T → F	F
8	F	F	F	~F → (F → ~F)	T → (F → T)	T → (T)	T	F	F → F	T

Summary truth table:

	$\sim p \to (q \to \sim r)$	q	$r \to p$	**Is it a counterexample row?**
1	T	T	T	○ yes ⊗ no
2	T	T	T	○ yes ⊗ no
3	T	F	T	○ yes ⊗ no
4	T	F	T	○ yes ⊗ no
5	F	T	F	○ yes ⊗ no
6	T	T	T	○ yes ⊗ no
7	T	F	F	○ yes ⊗ no
8	T	F	T	○ yes ⊗ no

④ The argument form is valid ⑤ because there are no counterexample rows in its truth table.

⑥ The argument is thus valid because its proper logical form is valid.

(d) If the US builds the wall on the Mexican border, then the illegal immigration problem will be solved. If the US builds the wall on the Mexican border, then the diplomatic relations between the USA and Mexico will suffer. So, if the illegal immigration problem will be solved then the diplomatic relations between the USA and Mexico will suffer.

① Symbolization:

B: USA builds the wall on the Mexican border
D: US-Mexico diplomatic relations will suffer.
I: Illegal immigration problem will be solved

$$\begin{array}{l} B \rightarrow I \\ \underline{B \rightarrow D} \\ I \rightarrow D \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} p \rightarrow q \\ \underline{p \rightarrow r} \\ q \rightarrow r \end{array}$$

③

	p	q	r	$p \rightarrow q$		$p \rightarrow r$		$q \rightarrow r$	
1	T	T	T	T → T	T	T → T	T	T → T	T
2	T	T	F	T → T	T	T → F	F	T → F	F
3	T	F	T	T → F	F	T → T	T	F → T	T
4	T	F	F	T → F	F	T → F	F	F → F	T
5	F	T	T	F → T	T	F → T	T	T → T	T
6	F	T	F	F → T	T	F → F	T	T → F	F
7	F	F	T	F → F	T	F → T	T	F → T	T
8	F	F	F	F → F	T	F → F	T	F → F	T

Summary truth table:

	$p \rightarrow q$	$p \rightarrow r$	$q \rightarrow r$	**Is it a counterexample row?**
1	T	T	T	○ yes ⊗ no
2	T	F	F	○ yes ⊗ no
3	F	T	T	○ yes ⊗ no
4	F	F	T	○ yes ⊗ no
5	T	T	T	○ yes ⊗ no
6	T	T	F	⊗ yes ○ no
7	T	T	T	○ yes ⊗ no
8	T	T	T	○ yes ⊗ no

④ The argument form is valid ⑤ because there are no counterexample rows in its truth table.

⑥ The argument is thus valid because its proper logical form is valid.

(e) If the US builds the wall on the Mexican border, then the illegal immigration problem will be solved provided that the immigrants do not find some other way to get into the US. However, the diplomatic relations between the USA and Mexico will suffer if the US builds the wall on the Mexican border. So, if the US builds the wall on the Mexican border, then the illegal immigration problem will be solved but the diplomatic relations between the USA and Mexico will suffer.

① Symbolization:

B: USA builds the wall on the Mexican border
D: US-Mexico diplomatic relations will suffer.
I: Illegal immigration problem will be solved
O: Immigrants find other way to get to US

$$\begin{array}{l} B \to (\sim O \to I) \\ \underline{B \to D} \\ B \to (I \bullet D) \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} p \to (\sim q \to r) \\ \underline{p \to s} \\ p \to (r \bullet s) \end{array}$$

	p	q	r	s	$p \to (\sim q \to r)$				$p \to s$		$p \to (r \bullet s)$		
1	T	T	T	T	$T \to (\sim T \to T)$	$T \to (F \to T)$	$T \to (T)$	T	$T \to T$	T	$T \to (T \bullet T)$	$T \to (T)$	T
2	T	T	T	F	$T \to (\sim T \to T)$	$T \to (F \to T)$	$T \to (T)$	T	$T \to F$	F	$T \to (T \bullet F)$	$T \to (F)$	F
3	T	T	F	T	$T \to (\sim T \to F)$	$T \to (F \to F)$	$T \to (T)$	T	$T \to T$	T	$T \to (F \bullet T)$	$T \to (F)$	F
4	T	T	F	F	$T \to (\sim T \to F)$	$T \to (F \to F)$	$T \to (T)$	T	$T \to F$	F	$T \to (F \bullet F)$	$T \to (F)$	F
5	T	F	T	T	$T \to (\sim F \to T)$	$T \to (T \to T)$	$T \to (T)$	T	$T \to T$	T	$T \to (T \bullet T)$	$T \to (T)$	T
6	T	F	T	F	$T \to (\sim F \to T)$	$T \to (T \to T)$	$T \to (T)$	T	$T \to F$	F	$T \to (T \bullet F)$	$T \to (F)$	F
7	T	F	F	T	$T \to (\sim F \to F)$	$T \to (T \to F)$	$T \to (F)$	F	$T \to T$	T	$T \to (F \bullet T)$	$T \to (F)$	F
8	T	F	F	F	$T \to (\sim F \to F)$	$T \to (T \to F)$	$T \to (F)$	F	$T \to F$	F	$T \to (F \bullet F)$	$T \to (F)$	F
9	F	T	T	T	$F \to (\sim T \to T)$	$F \to (F \to T)$	$F \to (T)$	T	$F \to T$	T	$F \to (T \bullet T)$	$F \to (T)$	T
10	F	T	T	F	$F \to (\sim T \to T)$	$F \to (F \to T)$	$F \to (T)$	T	$F \to F$	T	$F \to (T \bullet F)$	$F \to (F)$	T
11	F	T	F	T	$F \to (\sim T \to F)$	$F \to (F \to F)$	$F \to (T)$	T	$F \to T$	T	$F \to (F \bullet T)$	$F \to (F)$	T
12	F	T	F	F	$F \to (\sim T \to F)$	$F \to (F \to F)$	$F \to (T)$	T	$F \to F$	T	$F \to (F \bullet F)$	$F \to (F)$	T
13	F	F	T	T	$F \to (\sim F \to T)$	$F \to (T \to T)$	$F \to (T)$	T	$F \to T$	T	$F \to (T \bullet T)$	$F \to (T)$	T
14	F	F	T	F	$F \to (\sim F \to T)$	$F \to (T \to T)$	$F \to (T)$	T	$F \to F$	T	$F \to (T \bullet F)$	$F \to (F)$	T
15	F	F	F	T	$F \to (\sim F \to F)$	$F \to (T \to F)$	$F \to (F)$	T	$F \to T$	T	$F \to (F \bullet T)$	$F \to (F)$	T
16	F	F	F	F	$F \to (\sim F \to F)$	$F \to (T \to F)$	$F \to (F)$	T	$F \to F$	T	$F \to (F \bullet F)$	$F \to (F)$	T

Summary truth table:

	$(p \to (\sim q \to r)$	$p \to s$	$p \to (r \bullet s)$	**Is it a counterexample row?**
1	T	T	T	○ yes ⊗ no
2	T	F	F	○ yes ⊗ no
3	T	T	F	⊗ yes ○ no
4	T	F	F	○ yes ⊗ no
5	T	T	T	○ yes ⊗ no
6	T	F	F	○ yes ⊗ no
7	F	T	F	○ yes ⊗ no
8	F	F	F	○ yes ⊗ no
9	T	T	T	○ yes ⊗ no
10	T	T	T	○ yes ⊗ no
11	T	T	T	○ yes ⊗ no
12	T	T	T	○ yes ⊗ no
13	T	T	T	○ yes ⊗ no
14	T	T	T	○ yes ⊗ no
15	T	T	T	○ yes ⊗ no
16	T	T	T	○ yes ⊗ no

④ The argument form is invalid ⑤ because there is at least one counterexample row in its truth table, viz. row 3.

⑥ The argument is thus invalid because its proper logical form is invalid.

(f) If the universe is essentially deterministic then all of a person's actions can be predicted in advance. If people are entirely rational then all of a person's actions can be predicted in advance. Thus, if people are entirely rational then the universe is not essentially deterministic.

① Symbolization:

D: The universe is essentially deterministic.
P: All of a person's actions can be predicted.
R: People are entirely rational.

$$\begin{array}{l} D \rightarrow P \\ \underline{R \rightarrow P} \\ R \rightarrow \sim D \end{array}$$

② Proper logical form of the argument:

$$\begin{array}{l} p \rightarrow q \\ \underline{r \rightarrow q} \\ r \rightarrow \sim p \end{array}$$

③	p	q	r	$p \rightarrow q$		$r \rightarrow q$		$r \rightarrow \sim p$		
1	T	T	T	T → T	T	T → T	T	T → ~T	T → F	F
2	T	T	F	T → T	T	F → T	T	F → ~T	F → F	T
3	T	F	T	T → F	F	T → F	F	T → ~T	T → F	F
4	T	F	F	T → F	F	F → F	T	F → ~T	F → F	T
5	F	T	T	F → T	T	T → T	T	T → ~F	T → T	T
6	F	T	F	F → T	T	F → T	T	F → ~F	F → T	T
7	F	F	T	F → F	T	T → F	F	T → ~F	T → T	T
8	F	F	F	F → F	T	F → F	T	F → ~F	F → T	T

Summary truth table:

	$p \rightarrow q$	$r \rightarrow q$	$r \rightarrow \sim p$	**Is it a counterexample row?**
1	T	T	F	⊗ yes ○ no
2	T	T	T	○ yes ⊗ no
3	F	F	F	○ yes ⊗ no
4	F	T	T	○ yes ⊗ no
5	T	T	T	○ yes ⊗ no
6	T	T	T	○ yes ⊗ no
7	T	F	T	○ yes ⊗ no
8	T	T	T	○ yes ⊗ no

④ The argument form is invalid ⑤ because there is at least one counterexample row in its truth table, viz. row 1.

⑥ The argument is thus invalid because its proper logical form is invalid.

(g) You should file either tax form 12A or tax form 12C on this year's return only if you filed tax form 40B on your last year's return. You should file form 12A on this year's return only if your income was over $40000, and you should file tax form 12C only if your income did not exceed $40000. So, if your income exceeded $40000, then you must file tax form 12A.

① Symbolization:

A: You should file tax form 12A this year
C: You should file tax form 12C this year
B: you filed tax form 40B last year.
O: Your income exceeded $40000.

$$(A \vee C) \rightarrow B$$
$$\underline{(A \rightarrow O) \bullet (C \rightarrow \sim O)}$$
$$O \rightarrow A$$

② Proper logical form of the argument:

$$(p \vee q) \rightarrow r$$
$$\underline{(p \rightarrow s) \bullet (q \rightarrow \sim s)}$$
$$s \rightarrow p$$

	p	*q*	*r*	*s*	(*p* ∨ *q*) → *r*			(*p* → *s*) • (*q* → ~*s*)				*s* → *p*	
1	T	T	T	T	(T ∨ T) → T	(T) → T	T	(T → T) → (T → ~T)	(T) → (T → F)	T → (F)	F	T → T	T
2	T	T	T	F	(T ∨ T) → T	(T) → T	T	(T → F) → (T → ~F)	(F) → (T → T)	F → (T)	T	F → T	T
3	T	T	F	T	(T ∨ T) → F	(T) → F	F	(T → T) → (T → ~T)	(T) → (T → F)	T → (F)	F	T → T	T
4	T	T	F	F	(T ∨ T) → F	(T) → F	F	(T → F) → (T → ~F)	(F) → (T → T)	F → (T)	T	F → T	T
5	T	F	T	T	(T ∨ F) → T	(T) → T	T	(T → T) → (F → ~T)	(T) → (F → F)	T → (T)	T	T → T	T
6	T	F	T	F	(T ∨ F) → T	(T) → T	T	(T → F) → (F → ~F)	(F) → (F → T)	F → (T)	T	F → T	T
7	T	F	F	T	(T ∨ F) → F	(T) → F	F	(T → T) → (F → ~T)	(T) → (F → F)	T → (T)	T	T → T	T
8	T	F	F	F	(T ∨ F) → F	(T) → F	F	(T → F) → (F → ~F)	(F) → (F → T)	F → (T)	T	F → T	T
9	F	T	T	T	(F ∨ T) → T	(T) → T	T	(F → T) → (T → ~T)	(T) → (T → F)	T → (F)	F	T → F	F
10	F	T	T	F	(F ∨ T) → T	(T) → T	T	(F → F) → (T → ~F)	(T) → (T → T)	T → (T)	T	F → F	T
11	F	T	F	T	(F ∨ T) → F	(T) → F	F	(F → T) → (T → ~T)	(T) → (T → F)	T → (F)	F	T → F	F
12	F	T	F	F	(F ∨ T) → F	(T) → F	F	(F → F) → (T → ~F)	(T) → (T → T)	T → (T)	T	F → F	T
13	F	F	T	T	(F ∨ F) → T	(F) → T	T	(F → T) → (F → ~T)	(T) → (F → F)	T → (T)	T	T → F	F
14	F	F	T	F	(F ∨ F) → T	(F) → T	T	(F → F) → (F → ~F)	(T) → (F → T)	T → (T)	T	F → F	T
15	F	F	F	T	(F ∨ F) → F	(F) → F	T	(F → T) → (F → ~T)	(T) → (F → F)	T → (T)	T	T → F	F
16	F	F	F	F	(F ∨ F) → F	(F) → F	T	(F → F) → (F → ~F)	(T) → (F → T)	T → (T)	T	F → F	T

Summary truth table:

	$(p \lor q) \to r$	$(p \to s) \bullet (q \to \sim s)$	$s \to p$	**Is it a counterexample row?**
1	T	F	T	○ yes ⊗ no
2	T	T	T	○ yes ⊗ no
3	F	F	T	○ yes ⊗ no
4	F	T	T	○ yes ⊗ no
5	T	T	T	○ yes ⊗ no
6	T	T	T	○ yes ⊗ no
7	F	T	T	○ yes ⊗ no
8	F	T	T	○ yes ⊗ no
9	T	F	F	○ yes ⊗ no
10	T	T	T	○ yes ⊗ no
11	F	F	F	○ yes ⊗ no
12	F	T	T	○ yes ⊗ no
13	T	T	F	⊗ yes ○ no
14	T	T	T	○ yes ⊗ no
15	T	T	F	⊗ yes ○ no
16	T	T	T	○ yes ⊗ no

④ The argument form is invalid ⑤ because there is at least one counterexample row in its truth table, viz. rows 13 and 15.

⑥ The argument is thus invalid because its proper logical form is invalid.