Workbook Unit 7:

Validity and Invalidity: The Truth-Table Method

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Overview

In this unit, we will learn how to apply the truth-table method to check whether an argument form is valid or invalid.

This unit

- introduces the notion of an argument form
- teaches you how to fine the proper argument form of an argument
- introduces the concept of validity
- teaches how to determine whether an argument is valid by using the truth-table method

Prerequisites

There are three prerequisites for this unit. First, you need to be able to calculate the truth-value of an arbitrarily complex proposition without making any errors (Unit 4, Unit 5). Second, you need to be able to symbolize English sentences (Units 2, 3). Third, you need to be able to construct truth tables (Unit 6).

Basic Truth Tables (Reminder)

I know that you have filled out the basic truth tables many times. Do this once more if you had any calculation problems on the quizzes.

p	q	$p \bullet q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \equiv q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

p	$\sim p$
T	
F	

1. Arguments and Argument Forms

In Unit 6, we have introduced the distinction between propositional forms and propositions. We now need an analogous distinction between argument forms and arguments.

1.1. Proper Argument Forms

Consider the following arguments:

If **S**usan gets 100% on all quizzes, she will get an **A** in logic.
She did not get an **A**.
So, **S**usan did not get 100% on her quizzes.

$$\begin{array}{l} S \rightarrow A \\ \sim A \\ \hline \sim S \end{array}$$

If John **p**lants radishes in June, they will **g**row poorly.
The radishes did not **g**row poorly.
So, John did not **p**lant them in June.

$$\begin{array}{l} P \rightarrow G \\ \\ \hline \\ \end{array}$$

If it **r**ains, Patrick always takes an **u**mbrella with him.
Patrick did not take an **u**mbrella with him.
So, it did not **r**ain.

$$\begin{array}{l} R \rightarrow U \\ \\ \hline \\ \end{array}$$

These are different arguments though they share a common structure. We can represent the common structure in what is called the *proper argument form* of those arguments:

$$(\alpha)\quad \begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \sim p \end{array}$$

Complete the symbolizations of the above arguments to see that indeed all these arguments have the same argument form.

A proper argument form of an argument consists of propositional forms for all the premises and for the conclusion, where all the simple propositions that are the constituents of the premises and the conclusion are uniformly replaced with appropriate variables. Particular attention must be paid to preserve the distinction between the premises and the conclusion. Not only is the argument form (β) different from argument form (α),

$$(\beta)\quad \begin{array}{l} p \rightarrow q \\ \sim p \\ \hline \sim q \end{array}$$

unlike (α), argument form (β) is actually an invalid argument form as becomes evident when one considers the following substitution instance of (β):

If **S**usan gets 100% on all quizzes, she will get an **A** in logic.
Susan did not get 100% on her quizzes.
So, Susan did not get an **A**.

$$\begin{array}{l} S \rightarrow A \\ \\ \hline \\ \end{array}$$

This argument is invalid because from the fact that Susan did not get 100% on her quizzes, it does not follow that she did not get an A. She might have gotten 95% and still received an A – we simply cannot tell.

Note too that while the distinction between premises and the conclusion must be kept sharp – the order of the premises does *not* matter. So the following represent the same argument form:

(α)
$$\begin{array}{l} p \to q \\ \sim q \\ \hline \sim p \end{array}$$

(α′)
$$\begin{array}{l} \sim q \\ p \to q \\ \hline \sim p \end{array}$$

Exercise "Proper Argument Form" – 1

Provide the proper argument forms for the following arguments

(a)
$$\begin{array}{l} A \to B \\ A \\ \hline B \end{array}$$

(b)
$$\begin{array}{l} A \to B \\ B \\ \hline A \end{array}$$

(c)
$$\begin{array}{l} A \to B \\ \sim A \\ \hline \sim B \end{array}$$

(d)
$$\begin{array}{l} A \to B \\ \sim B \\ \hline \sim A \end{array}$$

(e)
$$\begin{array}{l} A \to B \\ C \to \sim B \\ C \\ \hline \sim A \end{array}$$

(f)
$$\begin{array}{l} \sim A \bullet \sim B \\ A \vee C \\ D \vee B \\ \hline D \end{array}$$

(g)
$$\begin{array}{l} (A \to B) \bullet (B \to A) \\ \hline A \equiv B \end{array}$$

(h)
$$\begin{array}{l} A \to B \\ B \to C \\ C \to A \\ \hline A \equiv C \end{array}$$

(i)
$$\begin{array}{l} (A \equiv B) \bullet \sim B \\ (C \equiv D) \bullet \sim D \\ \hline \sim A \bullet \sim C \end{array}$$

Exercise "Proper Argument Form" – 2

Connect the arguments (expressed in English) with their proper argument forms.

If Ann passes logic then she will either get married or she will find herself another boyfriend. Ann passed logic but did not get married. So, Ann found herself another boyfriend.	If Ann passes logic then she will not need both Bert's help and his love. Ann passed logic but she still needs Bert's love. So, she does not need his help.	If Ann does not pass logic, then she will need to retake it, but if she passes then she will not need to retake it. So, Ann will need to retake logic if and only if she does not pass it.	If Ann does not pass logic the first time or is very scared, then Bert will do all he can to help her. Bert did not need to help Ann. So, Ann passed logic and was not very scared.
$(\sim p \vee q) \rightarrow r$ $\sim r$ ――― $p \bullet \sim q$	$p \rightarrow (q \vee r)$ $p \bullet \sim q$ ――― r	$p \rightarrow \sim(q \bullet r)$ $p \bullet r$ ――― $\sim q$	$(\sim p \rightarrow q) \bullet (p \rightarrow \sim q)$ ――― $q \equiv \sim p$
If Daryl does not water her plants, they will wilt, but if she does water them they will not wilt. So, the plants will wilt if and only if Daryl does not water them.	If Ben will no longer fight with his sister or if he will improve at maths, he will get a hamster. Ben did not get a hamster. So, he kept fighting with his sister and did not improve at maths.	If Chris buys ice-cream then he will eat it either immediately or within an hour. He bought the ice-cream but did not have the time to eat it immediately. So, he ate it within an hour.	If *X* is an integer then it is not both odd and even. *X* is an integer and it is even. So *X* is not odd.

You can also do the exercise "Ex.07. Proper Argument Forms" on-line.

1.2. Validity, Invalidity, Argument Forms and Arguments

One reason why we need the notion of an argument form is that arguments are valid/invalid in virtue of their argument forms. This thought can be put more precisely in the form of the following statements:

> An argument is valid just in case its proper argument form is valid.
> An argument is invalid just in case its proper argument form is invalid.
> All substitution instances of a valid argument form are valid.
> All substitution instances of an invalid argument form are invalid.

(Do not worry for now if you don't see them as obvious. But you should worry if you still don't see them as obvious by the time you are ready to take the quiz.)

2. Invalidity

The intuitive idea behind validity is simple: an argument is valid just in case the conclusion follows logically from the premises. We have already tried to say what this means in Unit 1: an argument is valid just case if one accepts the premises one has to accept the conclusion, or, in other words, it is impossible for the conclusion to be false when the premises are true. All these formulations are to a greater or lesser extent intuitive, but it will really pay to spend some time trying to understand the notion of *in*validity first. Armed with a theoretical and practical grasp of invalidity we will then turn to validity.

2.1. Invalidity: Some Intuitions

An argument is invalid when the conclusion does not follow from the premises. Consider an example:

> If Susan gets 85 points on her quizzes, she will get a B.
> Susan got a B.
> So, she got 85 points her quizzes.

The conclusion just does not follow from the premises. It may be true that Susan got a B, and that she would get a B if she were to get 85 points on the quizzes, but it does not follow from these premises that she got 85 points. She may have gotten 84 or 86 points, for example, for which she would still have gotten a B.

Here is another example:

> If it rains then it is cloudy.
> It does not rain.
> So, it is not cloudy.

Here again the conclusion just does not follow from the premises. The premises may be true, yet the conclusion may be false! It is certainly true that if it rains then it is cloudy (the rain always falls from one cloud or another). It may be true that at a given point in Hattiesburg, say, it does not rain. Does it follow from this that it is not cloudy? No, it may happen that it does not rain though it is cloudy. In other words, it may happen that the premises are true and the conclusion is false. This makes the argument invalid.

2.2. Refutation by Counterexample

There is a useful technique of showing that an argument is invalid. It is called *refutation by counterexample*. In brief, the purpose of the technique is to show that an argument (the target argument) is invalid; the technique consists in presenting another argument, which has the same logical form as the target argument, in which the premises are clearly all true and the conclusion is clearly false.

Suppose that Bubba presents you with the following target argument, to which I will refer as the "JFK argument":

If John F. Kennedy was assassinated then he is dead.
John F. Kennedy is dead.
So, John F. Kennedy was assassinated.

☞ Think about this for a second. Is the argument valid? Or is it invalid?

You might be tempted to say that the argument is valid but in fact you should not! What you are most likely responding to is the truth of the conclusion. The conclusion is undoubtedly true but the argument is *not* valid because the conclusion does not follow from the premises.

To show that the conclusion does not follow from the premises in the JFK argument, we might argue like this:

> It is true that John F. Kennedy was assassinated, but the fact that he was assassinated does not follow from the fact that he is dead! There are people who are dead but who have not been assassinated. We cannot infer how they died from the mere fact that they are dead.

To make your point stronger (and in logical terms: to make it conclusive), you can produce an argument that has the same argument form, where it is clear that the premises are true and the conclusion false. Consider this argument:

If Ronald Reagan was assassinated then he is dead.
Ronald Reagan is dead.
So, Ronald Reagan was assassinated.

"Voila!", you say, and Bubba better hide where the sun don't shine for this is an argument with the very same logical form as the JFK argument but with true premises and a *false* conclusion!

You have basically employed here the technique of ***refutation by counterexample***. You have produced a counterexample to the validity of the JFK argument.

In order to refute an argument α by a counterexample you need to construct an argument β such that:

(a) argument β has the same argument form as argument α
(b) the premises of argument β are clearly and evidently true, while the conclusion of argument β is clearly and evidently false.

An argument that fulfills conditions (a) and (b) is called the counterexample argument – it is counterexample to the validity of the original argument.

(Note that I have here produced an argument instance that is close also thematically to the original argument. But this need not be the case. Any argument instance with true premises and a false conclusion (as long as it is of the same logical form) will show the original JFK argument to be invalid.)

Constructing real-life counterexamples is a skill one masters and it is very useful in all kinds of debates. As is usually the case in case of logic, mastering some logical techniques can actually help in developing the skill of producing real-life counterexample arguments.

It should also be noted that what is “clearly and evidently” true will depend on the state of knowledge of an audience, which means that when one wants to present a convincing counterexample one often has to take into account what the audience knows. It will not do to show off your knowledge of quantum mechanics and trying to produce a counterexample for an ordinary car mechanic.

Optional Exercise “Counterexamples”

Show that each of the following arguments is invalid by producing a counterexample argument that shares the logical form of the original argument, but where all the premises are evidently true while the conclusion is evidently false.
(Note that it may be very hard for you to come up with good examples. If it is, skip the exercise for now.)

(a) If Susan gets 92 points on her quizzes she will get an A in logic.
Susan got an A in logic.
____
So, Susan got 92 points on her quizzes.

(b) If Susan gets 92 points on her quizzes she will get an A in logic.
Susan did not get 92 points on her quizzes.
____
So, Susan did not get an A in logic

(c) If government spending increases then the economy will crash.
If unemployment rises then the economy will crash
____
So, if government spending increases then unemployment will rise.

2.3. Invalidity

The basic clue to the notion of invalidity is offered by the method of refutation by logical analogy. We can show that an argument α is invalid when we produce an argument β (of the same argument form as argument α), such that the premises of β are true and its conclusion is false. This generates our basic understanding of the notion of invalidity:

An argument invalid if and only if its proper argument form is invalid.
An argument form is invalid if and only if it has at least one substitution instance with true premises and a false conclusion.

As I said earlier, actually using the method of refutation by analogy in practice (using English sentences for the substitution of propositional variables) is a skill that you can acquire but it is difficult for most people. Moreover, there is no method associated with it that would guarantee your success in figuring out whether an argument is invalid. This has in part to do with the fact that there is an infinite number of substitution instances of any given argument form. Consider, for example, the argument form we have already encountered:

$$(\gamma) \quad \begin{array}{l} p \rightarrow q \\ q \\ \hline p \end{array}$$

There is an infinite number of substitution instances of this form, but among them will be arguments with true premises and false conclusion (so-called *counterexamples* to the validity of the argument form), which is enough to show that the argument form is invalid. A couple of such substitution instances of argument form γ are presented in Table 1.

If an argument $\mathcal{A}$ is a substitution instance of an argument form α then argument $\mathcal{A}$ is a ***counterexample*** to the validity of argument form α if and only if all the premises of argument $\mathcal{A}$ are true while its conclusion is false.

Table 1: A few substitution instances of the argument form γ.

Substitution Key	Premise: $p \to q$	Premise: q	Conclusion: p	
⋮	⋮	⋮	⋮	
p:=Marilyn Monroe had hair (was not bald) q:= MM had blond hair	If MM had blond hair then she had hair. ○ true ○ false	MM had hair ○ true ○ false	MM had blond hair. ○ true ○ false	☐ counterexample
p:=Danny de Vito has hair q:= DdV has blond hair	If DdV has blond hair then he has hair. ○ true ○ false	DdV has hair ○ true ○ false	DdV has blond hair. ○ true ○ false	☐ counterexample
⋮	⋮	⋮	⋮	
p:=JFK was assassinated q:=JFK is dead	If JFK was assassinated then he is dead ○ true ○ false	JFK is dead ○ true ○ false	JFK was assassinated ○ true ○ false	☐ counterexample
p:=Brad Pitt was assassi–nated q:=Brad Pitt is dead	If Brad Pitt was assassi–nated then he is dead ○ true ○ false	Brad Pitt is dead ○ true ○ false	Brad Pitt was assassi–nated ○ true ○ false	☐ counterexample
⋮	⋮	⋮	⋮	
p:=Lassie is a dog q:=Lassie is a mammal	If Lassie is a dog then Lassie is a mammal ○ true ○ false	Lassie is a mammal ○ true ○ false	Lassie is a dog ○ true ○ false	☐ counterexample
p:=Flipper is a dog q:=Flipper is a mammal	If Flipper is a dog then Flipper is a mammal ○ true ○ false	Flipper is a mammal ○ true ○ false	Flipper is a dog ○ true ○ false	☐ counterexample
⋮	⋮	⋮	⋮	
p:= q:=	○ true ○ false	○ true ○ false	○ true ○ false	☐ counterexample
p:= q:=	○ true ○ false	○ true ○ false	○ true ○ false	☐ counterexample
⋮	⋮	⋮	⋮	

Exercise "Invalidity"

(a) Add at least two more substitution instances of the argument form γ to Table 1.
(b) For each of the substitution instances shown in Table 1, decide the truth-value of the premises and the conclusion.
(b) Mark all the counterexample rows in Table 1.
(c) Explain why the argument form γ is invalid.
(d) Is the argument

If Lassie is a dog then Lassie is a mammal
Lassie is a mammal
____
Lassie is a dog

valid or invalid? Explain why.

2.4. Validity

If you *really* understand the notion of invalidity, you should understand the notion of validity, but be careful.

Validity is defined negatively thus:

An argument form is valid if and only if it is not invalid.

However, I should warn you at the very beginning that because the notion of validity is defined negatively, students usually have some problems in grasping it. So, be very careful here and later.

The basic definitions are simple:

An argument valid if and only if its proper argument form is valid.
An argument form is valid if and only if it has *no* substitution instances with true premises and a false conclusion.

Note that unlike in the case of an invalid argument form, it is enough to find at least one substitution instance that is a counterexample (has true premises and a false conclusion), in the case of a valid argument form we must make sure that it has no substitution instances that are counterexamples.

This means that in order to decide that an argument form is valid we have to check *all* of its substitution instances!!! Since there is an infinite number of substitution instances of any argument form (because there is an infinite number of English sentences – remember that English sentences can be very complex!), we cannot inspect all of them. This means that the basic definitions cited above render the notion of validity undecidable (for any given argument we could not determine that the argument is valid).

Fortunately, logicians have used the truth-table method to solve this problem once again (remember that there was a similar problem for the notion of tautology and that of contradiction).

3. Validity and Invalidity in terms of Truth Tables

Recall:

> An argument form is invalid if and only if it has at least one substitution instance with true premises and a false conclusion.
>
> An argument form is valid if and only if it has no substitution instances with true premises and a false conclusion.

The truth table method allows us to consider all substitution instances of an argument form in the aspect that matters, viz. the truth and falsehood of the premises and the conclusion. Thus, we can operationalize the concepts of validity and invalidity in terms of truth tables:

An argument form is **invalid** if and only there is **at least one row** of its truth table where its premises are true and the conclusion is false.
An argument form is **valid** if and only there is **no row** of its truth table where its premises are true and the conclusion is false.

We will soon learn how to construct a truth table for an argument form. To show that an argument form is invalid, it suffices to find *just one* row of the truth table where all the premises are true and the conclusion is false. To show that an argument form is valid, on the other hand, one must inspect *all* rows and find that in *none of them* all the premises are true while the conclusion is false.

We can formulate the definitions yet in another way using the notion of a counterexample row.

A row of a truth table for argument form α is a **counterexample row** (to the validity of the argument form α) if and only if all the premises are true in that row while the conclusion is false in it.

You can see that the notion of a counterexample row is analogical to the notion of a counterexample introduced in §2.3. With the notion of a counterexample row we can simplify our definition further:

An argument form α is **invalid** if and only there is **at least one counterexample row** in the truth table for α.
An argument form α is **valid** if and only there are **no counterexample rows** in the truth table for α.

Armed with these, we can proceed to learn how to apply them. We will do so in two stages. First (§3.1), we will learn to apply them "on dry runs" – by looking at possible truth tables. Second (§3.2), we will learn to construct truth tables for argument forms and decide whether argument forms are valid or not.

3.1. The Decision Procedures

Let us begin by looking at the decision procedures "on the dry," as it were – without yet looking at real arguments. The truth-table calculation will provide us with the truth-values for all the premises of an argument and its conclusion for all possible substitution instances. (We will such tables summary tables, see §3.2.)

Example 1

A completed truth table might look thus:

Row	Premise 1	Premise 2	Premise 3	Premise 4	Conclusion
1	F	F	F	T	F
2	T	T	T	T	T
3	T	T	T	T	F
4	T	F	T	F	T

This is a summary table for an argument with four premises, where the premises are complex statements, which involve only two propositional variables. (We know that the premises and the conclusion involve only two propositional variables because there are only four rows in the truth table.)

In order to decide whether an argument that has this truth-table is valid or not, we need to look for a counterexample row, i.e. we need to find a row where all the premises are true and the conclusion is false. If there is such a row, the argument form for which this is the truth table is invalid. If there is no such row, the argument form is valid. So, we need to answer the following questions:

- Is row 1 a counterexample row? ○ yes ○ no
- Is row 2 a counterexample row? ○ yes ○ no
- Is row 3 a counterexample row? ○ yes ○ no
- Is row 4 a counterexample row? ○ yes ○ no

☞ Try to answer the questions yourself before proceeding. Is the argument valid? Why?

In row 1, the conclusion is false, but not all the premises are true (only one of the premises is true), hence row 1 is not a counterexample row. In row 2, all of the premises are true but the conclusion is also true, so row 2 is not a counterexample to the validity of the argument form. In row 3, all of the premises are true and the conclusion is false. Row 3 is thus a counterexample to the validity of the argument. At this point, we need not look any further – we have sufficient evidence to judge that the argument form is invalid.

Just for the record, row 4 is likewise not a counterexample row – not all the premises are true in row 4 and the conclusion is not false in it.

Example 2

Consider another example – this time an argument with three premises where the premises and the conclusion involve three propositional variables (hence there are eight rows in the truth table):

Row	Premise 1	Premise 2	Premise 3	Conclusion
1	T	T	T	T
2	T	F	T	T
3	F	T	T	F
4	T	T	T	T
5	T	T	F	F
6	T	T	T	T
7	T	F	F	F
8	T	F	F	T

As before, we need to inspect the truth table by asking, this time, eight questions:

- Is row 1 a counterexample row? ○ yes ○ no
- Is row 2 a counterexample row? ○ yes ○ no
- Is row 3 a counterexample row? ○ yes ○ no
- Is row 4 a counterexample row? ○ yes ○ no
- Is row 5 a counterexample row? ○ yes ○ no
- Is row 6 a counterexample row? ○ yes ○ no
- Is row 7 a counterexample row? ○ yes ○ no
- Is row 8 a counterexample row? ○ yes ○ no

☞ Answer the questions yourself before proceeding. Is the argument valid? Why?

In row 1, all the premises are true but the conclusion is not false, so row 1 is not a counterexample row. In row 2, the conclusion is not false and not all the premises are true, so it is not a counterexample row. Row 3 is not a counterexample row because though the conclusion is false in it, not all the premises are true. In row 4, all the premises are true but the conclusion is not false, so row 4 is not a counterexample row. Row 5 is not a counterexample row because though the conclusion is false in it, not all the premises are true. In row 6, all the premises are true but the conclusion is not false, so row 6 is not a counterexample row. Row 7 is not a counterexample row because though the conclusion is false in it, not all the premises are true. In row 8, the conclusion is not false and not all the premises are true, so it is not a counterexample row.

Since there is no counterexample row, the argument is valid.

This brings up an important point about the determination of validity and invalidity.

To determine that an argument is valid, we need to inspect *all* the rows of the truth table and determine that they are *not* counterexample rows.

To determine that an argument is invalid, it suffices to find *one* counterexample row.

Note! The single most common error about validity that students make is to think that an argument form is valid if there is a row in its truth table with all premises true and a true conclusion. Nothing could be further from the concept of validity. (Check out row 2 in Example 1! The argument form in Example 1 is invalid but its truth table does have a row (row 2) where all premises are true and the conclusion is true too!) Validity requires that there be no row counterexample row. It does not actually even require, as we will see, that there be a row where all premises are true and a conclusion is true.

Example 3

Here is yet another truth table for an argument form with two premises, where the premises and the conclusion involve two propositional variables:

Row	Premise 1	Premise 2	Conclusion
1	F	T	F
2	T	F	T
3	T	F	F
4	T	F	T

Again, we need to ask four questions:

- Is row 1 a counterexample row? ○ yes ○ no
- Is row 2 a counterexample row? ○ yes ○ no
- Is row 3 a counterexample row? ○ yes ○ no
- Is row 4 a counterexample row? ○ yes ○ no

☞ Answer the questions yourself before proceeding. Is the argument valid? Why?

In row 1, the conclusion is false, but not all the premises are true, hence row 1 is not a counterexample row. In row 2, the conclusion is not false, so row 2 is not a counterexample row (in addition, not all the premises are true in that row). In row 3, the conclusion is false, but not all the premises are true, hence row 3 is not a counterexample row. In row 4, the conclusion is not false, so row 4 is not a counterexample row (in addition, not all the premises are true in that row).

Since there is no counterexample row in the truth table, we must conclude that the argument form for which this is a truth table is *valid*.

(Note that there is no row in this truth table for a valid argument form where all the premises are true and the conclusion is true. But that is all right. While the truth tables for some valid argument forms will have a row where all premises are true and the conclusion is true too, truth tables for other valid argument forms will have no such rows. As long as a truth table does not have any counterexample rows, the argument for which it is a truth table will be valid.)

Exercise "Validity – Dry"

Decide whether the argument form corresponding to the truth table you are given is valid or not.

(a)

Row	Premise 1	Premise 2	Conclusion	Is it a counterexample row?
1	F	T	T	○ yes (counterexample) ○ no
2	F	F	T	○ yes (counterexample) ○ no
3	T	T	F	○ yes (counterexample) ○ no
4	T	F	T	○ yes (counterexample) ○ no

The argument form for which this is a truth table is:

☐ valid because there are no counterexample rows

☐ invalid because there is at least one counterexample row (viz. row(s) ________).

(b)

Row	Premise 1	Premise 2	Conclusion	Is it a counterexample row?
1	F	T	F	○ yes (counterexample) ○ no
2	T	T	T	○ yes (counterexample) ○ no
3	T	F	F	○ yes (counterexample) ○ no
4	T	T	F	○ yes (counterexample) ○ no

The argument form for which this is a truth table is:

☐ valid because ____

☐ invalid because ____

(c)

Row	Prem. 1	Prem. 2	Prem. 3	Conclusion	Is it a counterexample row?
1	F	F	F	F	○ yes (counterexample) ○ no
2	T	F	F	T	○ yes (counterexample) ○ no
3	F	T	T	F	○ yes (counterexample) ○ no
4	T	F	F	T	○ yes (counterexample) ○ no

The argument form for which this is a truth table is:

☐ valid because ____

☐ invalid because ____

(d)

Row	Premise 1	Conclusion	Is it a counterexample row?
1	F	F	○ yes (counterexample) ○ no
2	F	T	○ yes (counterexample) ○ no
3	T	F	○ yes (counterexample) ○ no
4	T	T	○ yes (counterexample) ○ no

The argument form for which this is a truth table is:

☐ valid because ____

☐ invalid because ____

You can also do the exercise "Ex.07.Validity-on-the-Dry" on-line.

3.2. The Truth Table Method for Checking Validity

Example 1

Consider the following argument.

> If Jack gets 100 points on the test then he will not get a B for it.
> Jack got an B for the test.
> ____
> So, Jack did not get 100 points on the test.

It is a valid argument – the conclusion follows from the premises. Let us use the truth table method to demonstrate that this is indeed the case.

There are six steps involved in using the truth table method (they mirror the method used for determining whether a statement form is a tautology, a contradiction or a contingency):

1. Determine the logical form of the argument in question.

Calculation Table

2. Construct the base of the truth table.
3. Complete the construction of the truth table
4. Fill-in the truth table

Summary Table

5. Construct Summary Table
6. Determine whether the argument form is valid or invalid

For clarity, we will be constructing two truth tables: one calculation table, from which we will copy the results obtained to a summary table.

Step 1. Determining the Logical Form of the Argument

The logical form of the above argument is:

$$\begin{array}{l} p \rightarrow \sim q \\ q \\ \hline \sim p \end{array}$$

If you do not see it immediately, it will pay to first symbolize the argument. There are two simple propositions, for which we will need propositional constants:

J:= Jack gets 100 points on the test
B:= Jack gets a B for the test

We can then symbolize the argument accordingly:

$$\begin{array}{l} \mathrm{J} \rightarrow \sim \mathrm{B} \\ \mathrm{B} \\ \hline \sim \mathrm{J} \end{array}$$

You can see now that we have captured the logical form of this argument correctly.

Step 2. Constructing the Base of the Truth Table

We need to list alphabetically all the propositional variables occurring in all of the premises and the conclusion. In our case only two propositional variables occur in the premises and the conclusion, i.e., *p*, *q*. For the two variables above, our base of the truth table will consist of four rows.

p	*q*
T	T
T	F
F	T
F	F

Step 3. Complete the Construction of the Calculation Truth Table

Now you need to figure out is how all the possible combinations of the truth-values affect the truth-value of the premises and the conclusion. The truth table has to have separate columns for all the premises and the conclusion, thus:

		Premise 1	**Premise 2**	**Conclusion**
p	*q*	$p \rightarrow \sim q$	*q*	$\sim p$
T	T			
T	F			
F	T			
F	F			

Step 4: Fill-in the Calculation Truth Table

Now you are ready to fill in the truth table. Your task is to determine what the truth-value of each of the premises and the conclusion is in each row of the truth table. We are doing the same thing we have done so far except for multiple statements at the same time.

We can either work on all three statements at the same time or do them one by one. On the other examples, I will work on all the statements simultaneously. Here, let's do them one by one so that you realize just what's going on.

First, we substitute the relevant truth-values. CAUTION! Take your time to do this and double-check that you made the substitutions correctly. This is one place where errors often pop in.

p	*q*	$p \rightarrow \sim q$			*q*	$\sim p$	
T	T	T → ~T			T	~T	
T	F	T → ~F			F	~T	
F	T	F → ~T			T	~F	
F	F	F → ~F			F	~F	

☞ Try to do the calculation on your own. Is the argument valid? Why?

(A) To calculate the truth-value of the propositions, we will start with the first premise (marked by the brace).

p	q	$p \to \sim q$			q	$\sim p$
T	T	T → ~T	T → F	F	T	~T
T	F	T → ~F	T → T	T	F	~T
F	T	F → ~T	F → F	T	T	~F
F	F	F → ~F	F → T	T	F	~F

(B) There is no need in our case to calculate the second premise (marked by the brace) any further.

p	q	$p \to \sim q$			q	$\sim p$
T	T	T → ~T	T → F	F	T	~T
T	F	T → ~F	T → T	T	F	~T
F	T	F → ~T	F → F	T	T	~F
F	F	F → ~F	F → T	T	F	~F

(C) Conclusion (marked by the brace)

p	q	$p \to \sim q$			q	$\sim p$	
T	T	T → ~T	T → F	F	T	~T	F
T	F	T → ~F	T → T	T	F	~T	F
F	T	F → ~T	F → F	T	T	~F	T
F	F	F → ~F	F → T	T	F	~F	T

We have thus completed the calculation truth table. The columns that matter in the calculation table have been highlighted – these are the columns that show, for each of the premises and for the conclusion, whether the respective statements are true or false in a given row. However, to make things easier to visualize (which will be particularly helpful at quiz time), we will draw another table, the ***summary table***, in which we will put in just the truth-values of the whole statements.

Step 5: Summary Table

	p	q	$p \to \sim q$	q	$\sim p$
1	T	T	F	T	F
2	T	F	T	F	F
3	F	T	T	T	T
4	F	F	T	F	T

Step 6: Determine Whether the Argument Form Is Valid or Invalid

Looking at the Summary Table, we need to ask four questions:

- Is row 1 a counterexample row? ○ yes ○ no
- Is row 2 a counterexample row? ○ yes ○ no
- Is row 3 a counterexample row? ○ yes ○ no
- Is row 4 a counterexample row? ○ yes ○ no

☞ Is the argument valid? Why?

In row 1, although the conclusion is false, not all the premises are true, so row 1 is not a counterexample row. In row 2, the conclusion is false, but the premises are not both true. This is not a counterexample row. In row 3, all of the premises are true, but the conclusion is not false. Row 3 is not a counterexample row. In row 4, the premises are not both true and the conclusion is not false. This is not a counterexample row.

Since there is thus no counterexample row where all the premises are true and the conclusion false, the argument is valid. Our intuitions are thus confirmed.

Example 2

Let us use the truth-table method to decide whether the following argument is valid:

> Skinner's and Watson's theory are not both true.
> In fact, Watson's theory is false.
> ______________________________________
> So, Skinner's theory is true.

☞ What do your intuitions tell you about this argument? Does the conclusion follow from the premises? In other words, is it the case that someone who accepts the premises must accept the conclusion?

Well, let us apply our full-proof method of finding whether the argument is valid or not. To find out whether an argument is valid, we need to find out whether its proper argument form is valid. The proper argument form of this argument is:

$$\begin{array}{c} \sim p \lor \sim q \\ \sim q \\ \hline p \end{array} \qquad \text{or} \qquad \begin{array}{c} \sim(p \bullet q) \\ \sim q \\ \hline p \end{array}$$

(You can find the symbolization of the argument in the *Solutions*.) After finding an argument form, we then need to construct the truth table for this form. Our truth table will have four rows because two propositional variables *p* and *q* occur in the argument form. Remember that we need columns for each of the premises and for the conclusion. (You can check on your own that it does not matter which formulation we calculate, the summary table will be the same for both.)

	p	q	$\sim p \lor \sim q$			$\sim q$		p
1	T	T						
2	T	F						
3	F	T						
4	F	F						

☞ Complete the calculation truth table above and the summary table below and use the latter to determine whether the argument is valid or not.

	$\sim p \lor \sim q$	$\sim q$	p	**Is it a counterexample row?**
1				○ yes ○ no
2				○ yes ○ no
3				○ yes ○ no
4				○ yes ○ no

As you will see the argument form is in fact invalid – there is one counterexample row. You can see this intuitively, too, by thinking about the argument. If you know that Skinner's and Watson's theory are not both true, you also know that at least one of them is false, but this means that either only Watson's theory is false or only Skinner's theory is false or both are false. If you know further that Watson's theory is false you cannot infer anything definite about Skinner's theory – it may be true, but it might as well be false.

Exercise "Validity – 1"

Complete the truth tables to decide whether a given argument form is valid or not.

(a) $p \rightarrow q$
$\sim p$
______
$\sim q$

	p	q	$p \rightarrow q$		$\sim p$		$\sim q$	
1	T	T						
2	T	F						
3	F	T						
4	F	F						

Summary truth table:

	$p \rightarrow q$	$\sim p$	$\sim q$	**Is it a counterexample row?**
1				○ yes ○ no
2				○ yes ○ no
3				○ yes ○ no
4				○ yes ○ no

○ The argument form is valid because ____

○ The argument form is invalid because ____

(b) $p \equiv p$
______
p

	p	$p \equiv p$		p
1	T			
2	F			

Summary truth table:

	$p \equiv p$	p	**Is it a counterexample row?**
1			○ yes ○ no
2			○ yes ○ no

○ The argument form is valid because ____

○ The argument form is invalid because ____

(c) $p \to (q \to r)$
$p \to \sim q$
―――――――
$\sim r$

	p	q	r	$p \to (q \to r)$			$p \to \sim q$			$\sim r$	
1	T	T	T								
2	T	T	F								
3	T	F	T								
4	T	F	F								
5	F	T	T								
6	F	T	F								
7	F	F	T								
8	F	F	F								

Summary truth table:

	$p \to (q \to r)$	$p \to \sim q$	$\sim r$	**Is it a counterexample row?**
1				○ yes ○ no
2				○ yes ○ no
3				○ yes ○ no
4				○ yes ○ no
5				○ yes ○ no
6				○ yes ○ no
7				○ yes ○ no
8				○ yes ○ no

○ The argument form is valid because ______________________________

○ The argument form is invalid because ______________________________

(d) $p \equiv p$
―――――
$p \to p$

	p	$p \equiv p$		$p \to p$	
1	T				
2	F				

Summary truth table:

	$p \equiv p$	$p \to p$	**Is it a counterexample row?**
1			○ yes ○ no
2			○ yes ○ no

○ The argument form is valid because ______________________________

○ The argument form is invalid because ______________________________

(e) $p \vee q$
$\sim q$
---
p

	p	q	$p \vee q$		$\sim q$		p
1	T	T					
2	T	F					
3	F	T					
4	F	F					

Summary truth table:

	$p \vee q$	$\sim q$	p	**Is it a counterexample row?**
1				○ yes ○ no
2				○ yes ○ no
3				○ yes ○ no
4				○ yes ○ no

○ The argument form is valid because ____

○ The argument form is invalid because ____

(f) $p \rightarrow q$
$\sim q$

$\sim p$

	p	q	$p \rightarrow q$		$\sim q$		$\sim p$	
1	T	T						
2	T	F						
3	F	T						
4	F	F						

Summary truth table:

	$p \rightarrow q$	$\sim q$	$\sim p$	**Is it a counterexample row?**
1				○ yes ○ no
2				○ yes ○ no
3				○ yes ○ no
4				○ yes ○ no

○ The argument form is valid because ____

○ The argument form is invalid because ____

Exercise "Validity – 2"

Decide whether the following argument forms are valid or not. You can use the separate file with preprinted truth tables. Make sure you decide how many rows your truth table must have.

(a) $\dfrac{p \rightarrow q}{\sim p \rightarrow \sim q}$

(b) $\dfrac{p \rightarrow q}{\sim q \rightarrow \sim p}$

(c) $\dfrac{p}{p \bullet q}$

(d) $\dfrac{p}{p \vee q}$

(e) $\dfrac{p}{p \rightarrow q}$

(f) $\dfrac{p}{q \rightarrow p}$

(g)
$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline p \rightarrow r \end{array}$$

(h)
$$\begin{array}{l} p \rightarrow q \\ p \rightarrow r \\ \hline q \rightarrow r \end{array}$$

(i)
$$\begin{array}{l} p \rightarrow r \\ q \rightarrow r \\ \hline (p \vee q) \rightarrow r \end{array}$$

(j)
$$\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ p \vee r \\ \hline q \vee s \end{array}$$

(k)
$$\begin{array}{l} p \rightarrow q \\ (p \bullet q) \rightarrow r \\ p \rightarrow (r \rightarrow s) \\ \hline p \rightarrow s \end{array}$$

(l)
$$\begin{array}{l} p \rightarrow (q \rightarrow \mathrm{r}) \\ q \rightarrow (p \rightarrow \mathrm{s}) \\ \hline p \rightarrow s \end{array}$$

Exercise "Validity – 3"

Decide whether the following arguments are valid or not. You should ① symbolize the argument, ② present the proper logical form of the argument, ③ construct a truth table for the argument form, ④ decide whether the argument form is valid, ⑤ justify your decision, ⑥ decide whether the argument is valid.

(a) If Bob Woodward has the right sources then Henry Kissinger either advises the White House or is not retired. But Henry Kissinger is retired. So, Bob Woodward does not have the right sources.

(b) If Russia joins the European Union (the EU) then, if Rumania joins the EU then Bulgaria will join the EU. Rumania will join the EU. So if Bulgaria does not join the EU then Russia did not join the EU either.

(c) If Russia does not join the European Union (the EU) then, if Rumania joins the EU then Bulgaria will not join the EU. Rumania will join the EU. So if Bulgaria joins the EU then Russia will join the EU as well.

(d) If the US builds the wall on the Mexican border, then the illegal immigration problem will be solved. If the US builds the wall on the Mexican border, then the diplomatic relations between the USA and Mexico will suffer. So, if the illegal immigration problem will be solved then the diplomatic relations between the USA and Mexico will suffer.

(e) If the US builds the wall on the Mexican border, then the illegal immigration problem will be solved provided that the immigrants do not find some other way to get into the US. However, the diplomatic relations between the USA and Mexico will suffer if the US builds the wall on the Mexican border. So, if the US builds the wall on the Mexican border, then the illegal immigration problem will be solved but the diplomatic relations between the USA and Mexico will suffer.

(f) If people are entirely rational, then either all of a person's actions can be predicted in advance or the universe is essentially deterministic. Not all of a person's actions can be predicted in advance. Thus, if the universe is not essentially deterministic then people are not entirely rational. (after Copi & Cohen)

(g) You should file either tax form 12A or tax form 12C on this year's return only if you filed tax form 40B on your last year's return. You should file form 12A on this year's return only if your income was over $40000, and you should file tax form 12C only if your income did not exceed $40000. So, if your income exceeded $40000, then you must file tax form 12A.

What You Need to Know and Do

- You need to know what validity and invalidity is. What are some examples?
- When is an argument sound?
- You need to be able to use the truth table method to tell whether an argument form is valid or invalid.