Solutions to Workbook Exercises

Unit 6:

Tautologies, Contradictions and Contingencies

Exercise "Proper Logical Form" – 1

A → B	$p \rightarrow q$
A → (A → B)	$p \rightarrow (p \rightarrow q)$
(A → B) → A	$(p \rightarrow q) \rightarrow p$
(A ∨ B) → (~A ≡ ~B)	$(p \vee q) \rightarrow (\sim p \equiv \sim q)$
~(A • ~(B → ~~A))	$\sim(p \bullet \sim(q \rightarrow \sim\sim p))$
A	p

Exercise "Proper substitution instances"

The following are all proper substitution instances of the following propositional forms that use only the propositional constants: A, B, C.

$p \bullet \sim p$	A • ~A		B • ~B		C • ~C	
$p \rightarrow (p \equiv p)$	A → (A ≡ A)		B → (B ≡ B)		C → (C ≡ C)	
$p \rightarrow (q \vee p)$	A → (B ∨ A)	A → (C ∨ A)	B → (A ∨ B)	B → (C ∨ B)	C → (A ∨ C)	C → (B ∨ C)
$p \equiv (p \rightarrow q)$	A ≡ (A → B)	A ≡ (A → C)	B ≡ (B → A)	B ≡ (B → C)	C ≡ (C → A)	C ≡ (C → B)
$p \equiv (q \rightarrow q)$	A ≡ (B → B)	A ≡ (C → C)	B ≡ (A → A)	B ≡ (C → C)	C ≡ (A → A)	C ≡ (B → B)

$\sim(p \rightarrow q) \vee (\sim p \rightarrow \sim q)$	~(A → B) ∨ (~A → ~B)	~(A → C) ∨ (~A → ~C)
	~(B → A) ∨ (~B → ~A)	~(B → C) ∨ (~B → ~C)
	~(C → A) ∨ (~C → ~A)	~(C → B) ∨ (~C → ~B)
$\sim(p \rightarrow q) \vee (\sim q \rightarrow \sim p)$	~(A → B) ∨ (~B → ~A)	~(A → C) ∨ (~C → ~A)
	~(B → A) ∨ (~A → ~B)	~(B → C) ∨ (~C → ~B)
	~(C → A) ∨ (~A → ~C)	~(C → B) ∨ (~B → ~C)

drp@swps.edu.pl

Exercise “Proper Logical Form” – 2

Connect the propositions with their proper logical forms.

If Ann passes logic then she will either get married or she will find herself another boyfriend.	If Ann passes logic then she will need neither Bert’s help nor his love.	If Ann does not pass logic, then she will need to retake it, but if she passes then she will not need to retake it.	If Ann does not pass logic, then Bert will do all he can to help her; and, indeed, Ann will not pass logic.
$(\sim p \rightarrow q) \bullet (p \rightarrow \sim q)$	$p \rightarrow (q \vee r)$	$p \rightarrow (\sim q \bullet \sim r)$	$(\sim p \rightarrow q) \bullet \sim p$
If Chris buys ice-cream then he will buy neither cookies nor tea.	If Chris does not buy ice-cream then he will be miserable, but if he does buy ice-cream then he will not be miserable.	If Chris does not buy ice-cream then he will be miserable; in fact, Chris will not buy ice-cream.	If Chris buys ice-cream then he will eat it either immediately or within an hour.

If 10 is divisible by 5 then 10 is divisible by 1 and 2.	If 20 is divisible by 2 then if 40 is the result of multiplying 20 by 2 then 40 is divisible by 2 as well.	Either 10 is divisible by 2 or 10 is not divisible by 2 but by 1.	10 is divisible by 5 if and only if 10 is the product of 5 with some number.
$p \vee (\sim p \bullet q)$	$p \equiv q$	$p \rightarrow (q \bullet r)$	$p \rightarrow (q \rightarrow r)$
If you plant a seed then if you care for it properly, you will enjoy a beautiful plant.	If there is thunder, there is usually some heavy rain and hail.	Coffee is good if and only if it has half-&-half in it.	Chris will either get a dog, or he won’t get a dog but a cat.

Exercise Tautology—Contradiction—Contingency—1

Using the truth table method, determine whether a given propositional form is a tautology, a contradiction or a contingency. For each case, provide a *precise* justification.

(a)

p	$p \vee p$	
T	T ∨ T	**T**
F	F ∨ F	**F**

The propositional form $p \vee p$ is a contingency because there is at least one row where it is true (viz. row 1) and at least one row where it is false (viz. row 2).

(b)

p	$p \bullet p$	
T	T • T	**T**
F	F • F	**F**

The propositional form $p \bullet p$ is a contingency because there is at least one row where it is true (viz. row 1) and at least one row where it is false (viz. row 2).

(c)

p	$p \rightarrow p$	
T	T → T	**T**
F	F → F	**T**

The propositional form $p \rightarrow p$ is a tautology because it is true in all rows of the truth table

(d)

p	$p \equiv p$	
T	T ≡ T	**T**
F	F ≡ F	**T**

The propositional form $p \equiv p$ is a tautology because it is true in all rows of the truth table

(e)

p	$p \rightarrow {\sim}p$		
T	T → ~T	T → F	**F**
F	F → ~F	F → T	**T**

The propositional form $p \rightarrow {\sim}p$ is a contingency because there is at least one row where it is true (viz. row 2) and at least one row where it is false (viz. row 1).

(f)

p	${\sim}p \rightarrow p$		
T	~T → T	F → T	**T**
F	~F → F	T → F	**F**

The propositional form ${\sim}p \rightarrow p$ is a contingency because there is at least one row where it is true (viz. row 1) and at least one row where it is false (viz. row 2).

(g)

p	$p \equiv {\sim}p$		
T	T ≡ ~T	T ≡ F	**F**
F	F ≡ ~F	F ≡ T	**F**

The propositional form $p \equiv {\sim}p$ is a contradiction because it is false in all rows of its truth table.

(h)

p	${\sim}p \equiv p$		
T	~T ≡ T	F ≡ T	**F**
F	~F ≡ F	T ≡ F	**F**

The propositional form ${\sim}p \equiv p$ is a contradiction because it is false in all rows of its truth table.

(i)

p	$\sim(p \bullet \sim p)$			
T	~(T • ~T)	~(T • F)	~(F)	**T**
F	~(F • ~F)	~(F • T)	~(F)	**T**

The propositional form $\sim(p \bullet \sim p)$ is a tautology because it is true in all rows of its truth table.

(j)

p	$\sim(p \vee \sim p)$			
T	~(T ∨ ~T)	~(T ∨ F)	~(T)	**F**
F	~(F ∨ ~F)	~(F ∨ T)	~(T)	**F**

The propositional form $\sim(p \vee \sim p)$ is a contradiction because it is false in all rows of its truth table.

(k)

p	$\sim p$	
T	~T	**F**
F	~F	**T**

The propositional form $\sim p$ is a contingency because there is at least one row where it is true (viz. row 2) and at least one row where it is false (viz. row 1).

(l)

p	p
T	**T**
F	**F**

or

p
T
F

The propositional form p is a contingency because there is at least one row where it is true (viz. row 1) and at least one row where it is false (viz. row 2).

Exercise Tautology—Contradiction—Contingency—2

Using the truth table method, determine whether a given propositional form is a tautology, a contradiction or a contingency. For each case, provide a *precise* justification.

(a)

p	q	$(p \bullet q) \rightarrow p$		
T	T	(T • T) → T	(T) → T	**T**
T	F	(T • F) → T	(F) → T	**T**
F	T	(F • T) → F	(F) → F	**T**
F	F	(F • F) → F	(F) → F	**T**

The propositional form $(p \bullet q) \rightarrow p$ is a tautology because it is true in all rows of its truth table.

(b)

p	q	$p \rightarrow (p \bullet q)$		
T	T	T → (T • T)	T → (T)	**T**
T	F	T → (T • F)	T → (F)	**F**
F	T	F → (F • T)	F → (F)	**T**
F	F	F → (F • F)	F → (F)	**T**

The propositional form $p \rightarrow (p \bullet q)$ is a contingency because there is at least one row where it is true (viz. rows 1, 3, 4) and at least one row where it is false (viz. row 2).

(c)

p	q	$(p \vee q) \rightarrow p$		
T	T	(T ∨ T) → T	(T) → T	**T**
T	F	(T ∨ F) → T	(T) → T	**T**
F	T	(F ∨ T) → F	(T) → F	**F**
F	F	(F ∨ F) → F	(F) → F	**T**

The propositional form $(p \vee q) \rightarrow p$ is a contingency because there is at least one row where it is true (viz. rows 1, 2, 4) and at least one row where it is false (viz. row 3).

(d)

p	q	$p \rightarrow (p \vee q)$		
T	T	T → (T ∨ T)	T → (T)	**T**
T	F	T → (T ∨ F)	T → (T)	**T**
F	T	F → (F ∨ T)	F → (T)	**T**
F	F	F → (F ∨ F)	F → (F)	**T**

The propositional form $p \rightarrow (p \vee q)$ is a tautology because it is true in all rows of its truth table.

(e)

p	q	$(p \rightarrow q) \rightarrow (q \rightarrow p)$		
T	T	(T → T) → (T → T)	(T) → (T)	**T**
T	F	(T → F) → (F → T)	(F) → (T)	**T**
F	T	(F → T) → (T → F)	(T) → (F)	**F**
F	F	(F → F) → (F → F)	(T) → (T)	**T**

The propositional form $(p \rightarrow q) \rightarrow (q \rightarrow p)$ is a contingency because there is at least one row where it is true (viz. rows 1, 2, 4) and at least one row where it is false (viz. row 3).

(f)

p	q	$(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$			
T	T	(T → T) → (~T → ~T)	(T → T) → (F → F)	(T) → (T)	**T**
T	F	(T → F) → (~F → ~T)	(T → F) → (T → F)	(F) → (F)	**T**
F	T	(F → T) → (~T → ~F)	(F → T) → (F → T)	(T) → (T)	**T**
F	F	(F → F) → (~F → ~F)	(F → F) → (T → T)	(T) → (T)	**T**

The propositional form $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$ is a tautology because it is true in all rows of its truth table.

(g)

p	q	$p \equiv [p \vee (p \rightarrow q)]$			
T	T	T ≡ [T ∨ (T → T)]	T ≡ [T ∨ (T)]	T ≡ [T]	**T**
T	F	T ≡ [T ∨ (T → F)]	T ≡ [T ∨ (F)]	T ≡ [T]	**T**
F	T	F ≡ [F ∨ (F → T)]	F ≡ [F ∨ (T)]	F ≡ [T]	**F**
F	F	F ≡ [F ∨ (F → F)]	F ≡ [F ∨ (T)]	F ≡ [T]	**F**

The propositional form $p \equiv [p \vee (p \rightarrow q)]$ is a contingency because there is at least one row where it is true (viz. rows 1, 2) and at least one row where it is false (viz. rows 3, 4).

(h)

p	q	$[p \bullet (p \to q)] \to q$			
T	T	[T • (T → T)] → T	[T • (T)] → T	[T] → T	T
T	F	[T • (T → F)] → F	[T • (F)] → F	[F] → F	T
F	T	[F • (F → T)] → T	[F • (T)] → T	[F] → T	T
F	F	[F • (F → F)] → F	[F • (T)] → F	[F] → F	T

The propositional form $[p \bullet (p \to q)] \to q$ is a tautology because it is true in all rows of its truth table.

(i)

p	q	$[p \bullet (p \to q)] \to \sim q$			
T	T	[T • (T → T)] → ~T	[T • (T)] → F	[T] → F	F
T	F	[T • (T → F)] → ~F	[T • (F)] → T	[F] → T	T
F	T	[F • (F → T)] → ~T	[F • (T)] → F	[F] → F	T
F	F	[F • (F → F)] → ~F	[F • (T)] → T	[F] → T	T

The propositional form $[p \bullet (p \to q)] \to q$ a contingency because there is at least one row where it is true (viz. rows 2, 3, 4) and at least one row where it is false (viz. row 1).

(j)

p	q	$\sim\{[p \bullet (p \to q)] \to q\}$				
T	T	~{[T • (T → T)] → T}	~{[T • (T)] → T}	~{[T] → T}	~{T}	F
T	F	~{[T • (T → F)] → F}	~{[T • (F)] → F}	~{[F] → F}	~{T}	F
F	T	~{[F • (F → T)] → T}	~{[F • (T)] → T}	~{[F] → T}	~{T}	F
F	F	~{[F • (F → F)] → F}	~{[F • (T)] → F}	~{[F] → F}	~{T}	F

The propositional form $\sim\{[p \bullet (p \to q)] \to q\}$ is a contradiction because it is false in all rows of its truth table.

Exercise Tautology—Contradiction—Contingency—3

Using the truth table method, determine whether a given propositional form is a tautology, a contradiction or a contingency. For each case, provide a *precise* justification.

(a)

p	q	$\sim(p \bullet q)$		
T	T	~(T • T)	~(T)	F
T	F	~(T • F)	~(F)	T
F	T	~(F • T)	~(F)	T
F	F	~(F • F)	~(F)	T

The propositional form $\sim(p \bullet q)$ is a contingency because there is at least one row where it is true (viz. rows 2, 3, 4) and at least one row where it is false (viz. row 1).

(b)

p	q	$\sim p \vee \sim q$		
T	T	~T ∨ ~T	F ∨ F	F
T	F	~T ∨ ~F	F ∨ T	T
F	T	~F ∨ ~T	T ∨ F	T
F	F	~F ∨ ~F	T ∨ T	T

The propositional form $\sim p \vee \sim q$ is a contingency because there is at least one row where it is true (viz. rows 2, 3, 4) and at least one row where it is false (viz. row 1).

(c)

p	q	$(p \vee q) \rightarrow \sim p$		
T	T	(T ∨ T) → ~T	(T) → F	F
T	F	(T ∨ F) → ~T	(T) → F	F
F	T	(F ∨ T) → ~F	(T) → T	T
F	F	(F ∨ F) → ~F	(F) → T	T

The propositional form $(p \vee q) \rightarrow \sim p$ is a contingency because there is at least one row where it is true (viz. rows 3, 4) and at least one row where it is false (viz. rows 1, 2).

(d)

p	q	$\sim p \rightarrow (p \vee q)$		
T	T	~T → (T ∨ T)	F → (T)	T
T	F	~T → (T ∨ F)	F → (T)	T
F	T	~F → (F ∨ T)	T → (T)	T
F	F	~F → (F ∨ F)	T → (F)	F

The propositional form $\sim p \rightarrow (p \vee q)$ is a contingency because there is at least one row where it is true (viz. rows 1, 2, 3) and at least one row where it is false (viz. rows 4).

(e)

p	q	$(p \rightarrow q) \equiv (\sim p \vee q)$			
T	T	(T → T) ≡ (~T ∨ T)	(T) ≡ (F ∨ T)	(T) ≡ (T)	T
T	F	(T → F) ≡ (~T ∨ F)	(F) ≡ (F ∨ F)	(F) ≡ (F)	T
F	T	(F → T) ≡ (~F ∨ T)	(T) ≡ (T ∨ T)	(T) ≡ (T)	T
F	F	(F → F) ≡ (~F ∨ F)	(T) ≡ (T ∨ F)	(T) ≡ (T)	T

The propositional form $(p \rightarrow q) \equiv (\sim p \vee q)$ is a tautology because it is true in all rows of its truth table.

(f)

p	q	$\sim(p \rightarrow q) \equiv (\sim p \vee q)$			
T	T	~(T → T) ≡ (~T ∨ T)	~(T) ≡ (F ∨ T)	F ≡ (T)	F
T	F	~(T → F) ≡ (~T ∨ F)	~(F) ≡ (F ∨ F)	T ≡ (F)	F
F	T	~(F → T) ≡ (~F ∨ T)	~(T) ≡ (T ∨ T)	F ≡ (T)	F
F	F	~(F → F) ≡ (~F ∨ F)	~(T) ≡ (T ∨ F)	F ≡ (T)	F

The propositional form $\sim(p \rightarrow q) \equiv (\sim p \vee q)$ is a contradiction because it is false in all rows of its truth table.

(g)

p	q	$\sim(p \rightarrow q) \equiv (p \bullet \sim q)$			
T	T	~(T → T) ≡ (T • ~T)	~(T) ≡ (T • F)	F ≡ (F)	T
T	F	~(T → F) ≡ (T • ~F)	~(F) ≡ (T • T)	T ≡ (T)	T
F	T	~(F → T) ≡ (F • ~T)	~(T) ≡ (F • F)	F ≡ (F)	T
F	F	~(F → F) ≡ (F • ~F)	~(T) ≡ (F • T)	F ≡ (F)	T

The propositional form $\sim(p \rightarrow q) \equiv (p \bullet \sim q)$ is a tautology because it is true in all rows of its truth table.

(h)

p	q	$(p \to q) \to [p \vee (p \to q)]$			
T	T	$(T \to T) \to [T \vee (T \to T)]$	$T \to [T \vee (T)]$	$T \to [T]$	T
T	F	$(T \to F) \to [T \vee (T \to F)]$	$F \to [T \vee (F)]$	$F \to [T]$	T
F	T	$(F \to T) \to [F \vee (F \to T)]$	$T \to [F \vee (T)]$	$T \to [T]$	T
F	F	$(F \to F) \to [F \vee (F \to F)]$	$T \to [F \vee (T)]$	$T \to [T]$	T

The propositional form $(p \to q) \to [p \vee (p \to q)]$ is a tautology because it is true in all rows of its truth table.

(i)

p	q	$[p \vee (p \to q)] \to (p \to q)$			
T	T	$[T \vee (T \to T)] \to (T \to T)$	$[T \vee (T)] \to (T)$	$[T] \to T$	T
T	F	$[T \vee (T \to F)] \to (T \to F)$	$[T \vee (F)] \to (F)$	$[T] \to F$	F
F	T	$[F \vee (F \to T)] \to (F \to T)$	$[F \vee (T)] \to (T)$	$[T] \to T$	T
F	F	$[F \vee (F \to F)] \to (F \to F)$	$[F \vee (T)] \to (T)$	$[T] \to T$	T

The propositional form $[p \vee (p \to q)] \to (p \to q)$ is a contingency because there is at least one row where it is true (viz. rows 1, 3, 4) and at least one row where it is false (viz. row 2).

(j)

p	q	r	$[(p \to q) \bullet (q \to r)] \to (p \to r)$			
T	T	T	$[(T \to T) \bullet (T \to T)] \to (T \to T)$	$[(T) \bullet (T)] \to (T)$	$[T] \to (T)$	T
T	T	F	$[(T \to T) \bullet (T \to F)] \to (T \to F)$	$[(T) \bullet (F)] \to (F)$	$[F] \to (F)$	T
T	F	T	$[(T \to F) \bullet (F \to T)] \to (T \to T)$	$[(F) \bullet (T)] \to (T)$	$[F] \to (T)$	T
T	F	F	$[(T \to F) \bullet (F \to F)] \to (T \to F)$	$[(F) \bullet (T)] \to (F)$	$[F] \to (F)$	T
F	T	T	$[(F \to T) \bullet (T \to T)] \to (F \to T)$	$[(T) \bullet (T)] \to (T)$	$[T] \to (T)$	T
F	T	F	$[(F \to T) \bullet (T \to F)] \to (F \to F)$	$[(T) \bullet (F)] \to (T)$	$[F] \to (T)$	T
F	F	T	$[(F \to F) \bullet (F \to T)] \to (F \to T)$	$[(T) \bullet (T)] \to (T)$	$[T] \to (T)$	T
F	F	F	$[(F \to F) \bullet (F \to F)] \to (F \to F)$	$[(T) \bullet (T)] \to (T)$	$[T] \to (T)$	T

The propositional form $[(p \to q) \bullet (q \to r)] \to (p \to r)$ is a tautology because it is true in all rows of its truth table.

Exercise "Logical and Contingent Truth and Falsehood"

C: Cancun is the capital of the Mexico – false
E: El Dorado is the capital of the Mexico – false
M: Mexico City is the capital of Mexico – true
U: Washington, D.C. is the capital of the U.S.A. – true

(a) If Washington, D.C. is the capital of the U.S.A. then Mexico City is the capital of Mexico.

Symbolization:	$U \rightarrow M$
Truth-value calculation:	$T \rightarrow T$ T
Proper logical form:	$p \rightarrow q$
Truth-table:	see table below
The propositional form is:	a contingency
because:	there is at least one row where it is true (row 1, 3, 4) and there is at least one row where it is false (row 2)
The proposition is:	contingently true
because:	the proposition is true while its propositional form is a contingency

p	q	$p \rightarrow q$	
T	T	$T \rightarrow T$	**T**
T	F	$T \rightarrow F$	**F**
F	T	$F \rightarrow T$	**T**
F	F	$F \rightarrow F$	**T**

(b) If Washington, D.C. is the capital of the U.S.A. then Washington, D.C. is the capital of the U.S.A.

Symbolization:	$U \rightarrow U$
Truth-value calculation:	$T \rightarrow T$ T
Proper logical form:	$p \rightarrow p$
Truth-table:	see table below
The propositional form is:	a tautology
because:	it is true in all rows of its truth table.
The proposition is:	logically true
because:	its propositional form is a tautology.

p	$p \rightarrow p$	
T	$T \rightarrow T$	**T**
F	$F \rightarrow F$	**T**

(c) If Cancun is the capital of Mexico then Cancun is the capital of Mexico.

Symbolization:	C → C
Truth-value calculation:	F → F T
Proper logical form:	*p* → *p*
Truth-table:	see table below
The propositional form is:	a tautology
because:	it is true in all rows of its truth table.
The proposition is:	logically true
because:	its propositional form is a tautology.

p	*p* → *p*	
T	T → T	T
F	F → F	T

(d) Either Cancun is the capital of Mexico or Cancun is the capital of Mexico.

Symbolization:	C ∨ C
Truth-value calculation:	F ∨ F F
Proper logical form:	*p* ∨ *p*
Truth-table:	see table below
The propositional form is:	a contingency
because:	there is at least one row where it is true (row 1) and there is at least one row where it is false (row 2)
The proposition is:	contingently false
because:	the proposition is false while its propositional form is a contingency

p	*p* ∨ *p*	
T	T ∨ T	T
F	F ∨ F	F

(e) Either Cancun is the capital of Mexico or Cancun is not the capital of Mexico.

Symbolization:	C ∨ ~C
Truth-value calculation:	F ∨ ~F T
Proper logical form:	*p* ∨ ~*p*
Truth-table:	see table below
The propositional form is:	a tautology
because:	it is true in all rows of its truth table.
The proposition is:	logically true
because:	its propositional form is a tautology.

p	*p* ∨ ~*p*		
T	T ∨ ~T	T ∨ F	T
F	F ∨ ~F	F ∨ T	T

(f) If either Cancun or Mexico City is the capital of Mexico then Cancun is the capital of Mexico.

Symbolization:	$(C \vee M) \rightarrow C$
Truth-value calculation:	$(F \vee T) \rightarrow F$ $(T) \rightarrow F$ F
Proper logical form:	$(p \vee q) \rightarrow p$
Truth-table:	see table below
The propositional form is:	a contingency
because:	there is at least one row where it is true (row 1, 2, 4) and there is at least one row where it is false (row 3)
The proposition is:	contingently false
because:	the proposition is false while its propositional form is a contingency

p	q	$(p \vee q) \rightarrow p$		
T	T	$(T \vee T) \rightarrow T$	$(T) \rightarrow T$	T
T	F	$(T \vee F) \rightarrow T$	$(T) \rightarrow T$	T
F	T	$(F \vee T) \rightarrow F$	$(T) \rightarrow F$	F
F	F	$(F \vee F) \rightarrow F$	$(F) \rightarrow F$	T

(g) If Cancun is the capital of Mexico then either Cancun or El Dorado is the capital of Mexico.

Symbolization:	$C \rightarrow (C \vee E)$
Truth-value calculation:	$F \rightarrow (F \vee F)$ $F \rightarrow (F)$ T
Proper logical form:	$p \rightarrow (p \vee q)$
Truth-table:	see table below
The propositional form is:	a tautology
because:	it is true in all rows of its truth table.
The proposition is:	logically true
because:	its propositional form is a tautology.

p	q	$p \rightarrow (p \vee q)$		
T	T	$T \rightarrow (T \vee T)$	$T \rightarrow (T)$	T
T	F	$T \rightarrow (T \vee F)$	$T \rightarrow (T)$	T
F	T	$F \rightarrow (F \vee T)$	$F \rightarrow (T)$	T
F	F	$F \rightarrow (F \vee F)$	$F \rightarrow (F)$	T

(h) If Mexico City is the capital of Mexico then either Cancun or Mexico City is the capital of Mexico.

Symbolization:	$M \rightarrow (C \vee M)$
Truth-value calculation:	$T \rightarrow (F \vee T)$ $T \rightarrow (T)$ T
Proper logical form:	$p \rightarrow (q \vee p)$
Truth-table:	see table below
The propositional form is:	a tautology
because:	it is true in all rows of its truth table.
The proposition is:	logically true
because:	its propositional form is a tautology.

p	q	$p \rightarrow (q \vee p)$		
T	T	$T \rightarrow (T \vee T)$	$T \rightarrow (T)$	T
T	F	$T \rightarrow (F \vee T)$	$T \rightarrow (T)$	T
F	T	$F \rightarrow (T \vee F)$	$F \rightarrow (T)$	T
F	F	$F \rightarrow (F \vee F)$	$F \rightarrow (F)$	T

(i) Neither Cancun nor El Dorado are the capital of Mexico.

Symbolization:	$\sim C \bullet \sim E$
Truth-value calculation:	$\sim F \bullet \sim F$ $T \bullet T$ T
Proper logical form:	$\sim p \bullet \sim q$
Truth-table:	see table below
The propositional form is:	a contingency
because:	there is at least one row where it is true (row 4) and there is at least one row where it is false (row 1, 2, 3)
The proposition is:	contingently true
because:	the proposition is true while its propositional form is a contingency

p	q	$\sim p \bullet \sim q$		
T	T	$\sim T \bullet \sim T$	$F \bullet F$	F
T	F	$\sim T \bullet \sim F$	$F \bullet T$	F
F	T	$\sim F \bullet \sim T$	$T \bullet F$	F
F	F	$\sim F \bullet \sim F$	$T \bullet T$	T

(j) Neither Cancun nor Mexico City are the capital of Mexico.

Symbolization:	~C • ~M
Truth-value calculation:	~F • ~T T • F F
Proper logical form:	*~p* • *~q*
Truth-table:	see table below
The propositional form is:	a contingency
because:	there is at least one row where it is true (row 4) and there is at least one row where it is false (row 1, 2, 3)
The proposition is:	contingently false
because:	the proposition is false while its propositional form is a contingency

p	*q*	*~p* • *~q*		
T	T	~T • ~T	F • F	**F**
T	F	~T • ~F	F • T	**F**
F	T	~F • ~T	T • F	**F**
F	F	~F • ~F	T • T	**T**

(k) Either Cancun or Mexico City are the capital of Mexico, but not both.

Symbolization:	(C ∨ M) • ~(C • M)
Truth-value calculation:	(F ∨ T) • ~(F • T) (T) • ~(F) T • T T
Proper logical form:	(*p* ∨ *q*) • ~(*p* • *q*)
Truth-table:	see table below
The propositional form is:	a contingency
because:	there is at least one row where it is true (row 2, 3) and there is at least one row where it is false (row 1, 4)
The proposition is:	contingently true
because:	the proposition is true while its propositional form is a contingency

p	*q*	(*p* ∨ *q*) • ~(*p* • *q*)			
T	T	(T ∨ T) • ~(T • T)	(T) • ~(T)	T • F	**F**
T	F	(T ∨ F) • ~(T • F)	(T) • ~(F)	T • T	**T**
F	T	(F ∨ T) • ~(F • T)	(T) • ~(F)	T • T	**T**
F	F	(F ∨ F) • ~(F • F)	(F) • ~(F)	F • T	**F**

(l) Cancun is the capital of Mexico, however, neither El Dorado nor Cancun is the capital of Mexico.

Symbolization:	C • (~E • ~C)
Truth-value calculation:	F • (~F • ~F) F • (T • T) F • (T) F
Proper logical form:	*p* • (~*q* • ~*p*)
Truth-table:	see table below
The propositional form is:	a contradiction
because:	it is false in all rows of its truth table.
The proposition is:	logically false
because:	its propositional form is a contradiction.

p	*q*	*p* • (~*q* • ~*p*)			
T	T	T • (~T • ~T)	T • (F • F)	T • (F)	F
T	F	T • (~F • ~T)	T • (T • F)	T • (F)	F
F	T	F • (~T • ~F)	F • (F • T)	F • (F)	F
F	F	F • (~F • ~F)	F • (T • T)	F • (T)	F

(m) Cancun is not the capital of Mexico while El Dorado is the capital of Mexico.

Symbolization:	~C • E
Truth-value calculation:	~F • F T • F F
Proper logical form:	~*p* • *q*
Truth-table:	see table below
The propositional form is:	a contingency
because:	there is at least one row where it is true (row 3) and there is at least one row where it is false (rows 1, 2, 4)
The proposition is:	contingently false
because:	the proposition is false while its propositional form is a contingency

p	*q*	~*p* • *q*		
T	T	~T • T	F • T	F
T	F	~T • F	F • F	F
F	T	~F • T	T • T	T
F	F	~F • F	T • F	F

(n) Cancun is the capital of Mexico provided that El Dorado is not the capital of Mexico.

Symbolization:	~E → C
Truth-value calculation:	~F → F T → F F
Proper logical form:	~*p* → *q*
Truth-table:	see table below
The propositional form is:	a contingency
because:	there is at least one row where it is true (row 1, 2, 3) and there is at least one row where it is false (rows 4)
The proposition is:	contingently false
because:	the proposition is false while its propositional form is a contingency

p	*q*	~*p* → *q*		
T	T	~T → T	F → T	T
T	F	~T → F	F → F	T
F	T	~F → T	T → T	T
F	F	~F → F	T → F	F

(o) Either Cancun is the capital of Mexico just in case Cancun is not the capital of Mexico or El Dorado is the capital of Mexico just in case El Dorado is not the capital of Mexico.

Symbolization:	(C ≡ ~C) ∨ (E ≡ ~E)
Truth-value calculation:	(F ≡ ~F) ∨ (F ≡ ~F) (F ≡ T) ∨ (F ≡ T) (F) ∨ (F) F
Proper logical form:	(*p* ≡ ~*p*) ∨ (*q* ≡ ~*q*)
Truth-table:	see table below
The propositional form is:	a contradiction
because:	it is false in all rows of its truth table.
The proposition is:	logically false
because:	its propositional form is a contradiction.

p	*q*	(*p* ≡ ~*p*) ∨ (*q* ≡ ~*q*)			
T	T	(T ≡ ~T) ∨ (T ≡ ~T)	(T ≡ F) ∨ (T ≡ F)	(F) ∨ (F)	F
T	F	(T ≡ ~T) ∨ (F ≡ ~F)	(T ≡ F) ∨ (F ≡ T)	(F) ∨ (F)	F
F	T	(F ≡ ~F) ∨ (T ≡ ~T)	(F ≡ T) ∨ (T ≡ F)	(F) ∨ (F)	F
F	F	(F ≡ ~F) ∨ (F ≡ ~F)	(F ≡ T) ∨ (F ≡ T)	(F) ∨ (F)	F