Workbook Unit 6:

Tautologies, Contradictions and Contingencies

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Overview

In this unit, we will put the skills of truth-value calculations into action. You will learn about certain properties of propositions (and about certain properties of propositional forms), and you will learn how to determine which of the properties propositions have using the technique of constructing truth tables.

This unit

- introduces the distinction between propositions and propositional forms
- introduces the concept of a proper propositional form of a proposition and the concept of a proper substitution instance of a propositional form
- introduces four properties of propositions – logical truth, contingent truth, logical falsehood and contingent falsehood
- introduces three properties of propositional forms – tautologousness, contingency, contradictoriness
- relates the properties of propositions with those of propositional forms
- teaches you to how to construct a truth table for a propositional form
- teaches you how to determine whether a propositional form is a tautology, a contingency or a contradiction
- teaches you how to determine whether a proposition is logically true, contingently true, logically false or contingently false.

PowerPoint Presentation

There is a PowerPoint Presentation that accompanies this Unit. It is available on-line as a .pps and a zipped .pps file. The presentation is particularly useful for understanding how to construct a truth-table base.

Prerequisites

There are two prerequisites for this unit. First, you need to be able to calculate the truth-value of an arbitrarily complex proposition without making any errors (Unit 4, Unit 5). Second, you need to be able to symbolize English sentences (Units 2, 3).

Basic Truth Tables (Reminder)

I know that you have filled out the basic truth tables many times. Do this once more if you had any calculation problems on the quizzes.

p	q	$p \bullet q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \equiv q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

p	$\sim p$
T	
F	

1. Four Properties of Propositions

1.1. Contingent and Logical Truth

Consider the following examples of statements:

- The President of the U.S.A. is a man.
- Some students are bored by logic.
- Some roses are red.
- Washington, D.C. is the capital of the U.S.A

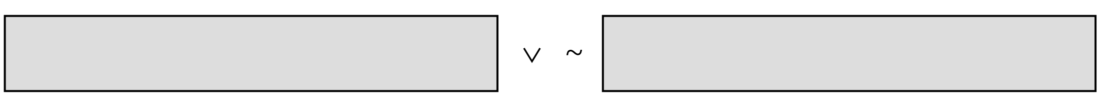
- It will either rain or not rain in Hattiesburg on June 28, 2020
- Either all students like logic or some students don't like logic.
- If I love you then I love you.

All these statements are true but there is a marked difference between the propositions in the left and right column. It is hard for us to say exactly how they differ – we certainly don't want to say that the propositions on the right are "more true" than the propositions on the left (the property of truth does not admit of degrees). To capture the difference, logicians have introduced the distinction between *logical truth* and *contingent truth*.

The truth of contingently true statements (on the left) depends on how things are in the world. Some roses are, *as a matter of fact*, red, but you could easily imagine that there would be no red roses just as there are no red sunflowers. The President of the U.S.A. is, *as a matter of fact*, a man but facts could be otherwise in this case as well. All these statements are contingently true – they are true contingent on the facts.

The statements on the right are *logically* true, on the other hand. They are true necessarily, independently of the facts. The claim "It will either rain or not rain in Hattiesburg on June 28, 2020" is true independently of how the facts turn out. If on June 28, 2020 it does in fact rain, then the statement "It will either rain or not rain in Hattiesburg on June 28, 2020" will be true. However, if it so happens that on June 28, 2020 it does not rain, then the statement "It will either rain or not rain in Hattiesburg on June 28, 2020" will still be true.

Logically true propositions are true *in virtue of their logical form*. This means that any proposition that shares the same logical form with a given logically true proposition will also be true, in fact will be logically true. The logical form of the proposition "It will either rain or not rain in Hattiesburg on June 28, 2020" can be captured thus:



where any proposition can jump into the boxes (as long as the same proposition jumps into both boxes). I have already said earlier that since logicians are lazy and don't like to draw boxes into which propositions jump, they have invented the idea of a propositional variable, which is a symbol that stands in for such a box. The logical form of the proposition "It will either rain or not rain in Hattiesburg on June 28, 2020" can thus be written:

$$p \vee {\sim}p$$

As you will learn later, the propositional form *p* ∨ ~*p* is called a tautology. In fact, the logical forms of logically true propositions are tautologous. We will return to this later since I want you to understand the distinction between these properties – the property of logical truth (which pertains to propositions) and the property of tautologousness (which pertains to propositional forms).

1.2. Contingent and Logical Falsehood

Consider the following examples of statements:

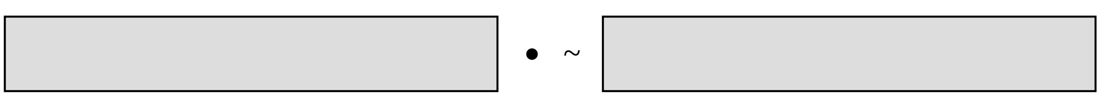
- The President of the U.S.A. is a woman.
- Nobody likes logic.
- Some roses smell like dirty socks.
- The President of Poland is open-minded

- It will neither rain nor not rain in Hattiesburg on June 28, 2020
- Betty will marry John but she will not marry him
- I love you just in case I don't love you.

While all these statements are false, as before, we feel that they are differently false. Here again logicians help by insisting that while the statements on left are contingently false, the statements on the right are logically false. Contingently false statements are false because of how things are in the world. As a matter of fact, no roses smell like dirty socks, but you could easily imagine that there would some that do (there are after all flowers, even edible fruit, that do smell awful). The President of Poland likewise could be open-minded and tolerant. All these statements are contingently false – they are false contingent on the facts.

The statements on the right are *logically* false, on the other hand. They are false necessarily, independently of the facts. The claim "Betty will marry John but she will not marry him" is false independently of how the facts turn out. If Betty does in fact marry John, then the statement "Betty will marry John but she will not marry him" will be false. And if Betty does not in fact marry John, the statement "Betty will marry John but she will not marry him" will still be false.

Logicians say that logically false propositions are false *in virtue of their logical form*. This means roughly that any proposition that shares the same logical form with a given logically false proposition will also be logically false. The logical form of the proposition "Betty will marry John but she will not marry him" can be captured thus:



where again any proposition can jump into the boxes (as long as the same proposition jumps into both boxes). The logical form of the proposition "Betty will marry John but she will not marry him" can also be written as:

$p \bullet {\sim}p$

As you will learn later, the propositional form *p* • ~*p* is called a contradiction. In fact, the logical forms of logically false propositions are contradictory.

2. Propositional Forms

In Unit 1, we have introduced the concept of a proposition as the proper bearer of truth-values. We have said that unambiguous declarative sentences express propositions. You have just seen in §1 that there are reasons to speak of the logical form of propositions. This is why logicians have introduced the notion of a propositional form.

2.1. Proper Logical Form of a Proposition

Consider the following sentence:

(1) If Betty passes logic then she will get a car, but if she does not then she will not get it.

Thus far, when faced with such a sentence, we have learned how to symbolize it, i.e. how to represent it by means of logical constants (connectives, parentheses) and propositional constants, to which we have been assigning simple sentences (in English). A symbolization of the sentence (1) would look thus:

L: Betty passes logic
C: Betty will get a car

[1] $(L \rightarrow C) \bullet (\sim L \rightarrow \sim C)$

Consider another sentence:

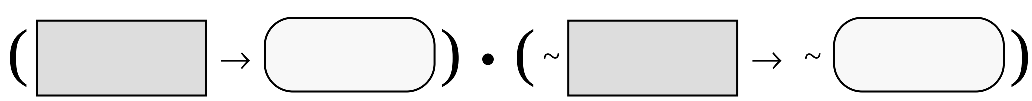
(2) If platypus are mammals then platypus babies are nursed by their mothers, but if platypus are not mammals then platypus babies are not nursed by their mothers.

Again we could symbolize sentence (2) thus:

M: Platypus are mammals
N: Platypus babies are nursed by their mothers

[2] $(M \rightarrow N) \bullet (\sim M \rightarrow \sim N)$

These are different propositions but they share their logical structure. The logical structure of those propositions can be represented by means of boxes thus:



(Note that we need two types of boxes because there are two different simple propositions.) The logical structure of these propositions can also be represented by means of variables thus:

(i) $(p \rightarrow q) \bullet (\sim p \rightarrow \sim q)$

Formula (i) is also called ***a propositional form***. A propositional form is composed out of logical constants (connectives, parentheses) according to the same composition rules as propositions (i.e. there may only be one main connective, the left and right parentheses must match, etc.), but there are no propositional constants (A, B, C, etc.) in propositional forms – there are only propositional variables (*p*, *q*, *r*, etc.).

At this point, I'm sure you can't help but feel that there is no real difference between (1) or [1], on the one hand, and (i), on the other – but the difference is very real, moreover, this difference (and your proper understanding of it) will be crucial for your understanding the topic. This is why we need to spend a little more time thinking about propositional forms (see §2.2).

Formula (i) is a propositional form, while statement (1) and its symbolization [1] is a proposition. But clearly there is more to say about the relationship between (i) and (1)/[1]. Intuitively, we want to say that formula (i) captures the logical structure of proposition (1)/[1]. More formally, we will say that propositional form (i) is the ***proper logical form*** (in propositional logic) of proposition (1)/[1]. And conversely, proposition (1)/[1] is a ***proper substitution instance*** of propositional form (i).

Definition

A proposition $\mathcal{P}$ is a **proper substitution instance** of a propositional form $\mathcal{F}$ iff $\mathcal{F}$ and $\mathcal{P}$ identical except that that propositional variables in $\mathcal{F}$ have been replaced with propositional constants in $\mathcal{P}$, where (a) same-shaped propositional variables were replaced with same-shaped propositional constants and (b) differently-shaped propositional variables were replaced with differently shaped propositional constants.

Definition

Propositional form $\mathcal{F}$ is a **proper logical form** of proposition $\mathcal{P}$ iff a proposition $\mathcal{P}$ is a proper substitution instance of a propositional form $\mathcal{F}$.

A given propositional form

(i) $(p \rightarrow q) \bullet (\sim p \rightarrow \sim q)$

is thus the proper logical form of the following propositions:

[1] $(L \rightarrow C) \bullet (\sim L \rightarrow \sim C)$
[2] $(M \rightarrow N) \bullet (\sim M \rightarrow \sim N)$
[3] $(A \rightarrow B) \bullet (\sim A \rightarrow \sim B)$

but it is *not* the proper logical form of the following propositions:

[4]	$(L \vee C) \bullet (\sim L \vee \sim C)$	because (i) and [4] differ not only in that where there are propositional constants in [4] there are propositional variables in (i); the logical constants differ as well!
[5]	$(C \rightarrow D) \bullet (\sim D \rightarrow \sim C)$	because condition (a) is not satisfied
[6]	$(A \rightarrow A) \bullet (\sim A \rightarrow \sim A)$	because condition (b) is not satisfied

Exercise “Proper Logical Form” – 1

Provide the proper logical forms for the following propositions

A → B	$p \rightarrow q$
A → (A → B)	$p \rightarrow (p \rightarrow q)$
(A → B) → A	$(p \rightarrow q) \rightarrow p$
(A ∨ B) → (~A ≡ ~B)	$(p \vee q) \rightarrow (\sim p \equiv \sim q)$
~(A • ~(B → ~~A))	$\sim(p \bullet \sim(q \rightarrow \sim\sim p))$
A	p

Exercise “Proper substitution instances”

Provide at least two different examples of proper substitution instances of the following propositional forms (use only the following propositional constants: A, B, C)

$p \bullet \sim p$		
$p \rightarrow (p \equiv p)$		
$p \rightarrow (q \vee p)$		
$p \equiv (p \rightarrow q)$		
$p \equiv (q \rightarrow q)$		
$\sim(p \rightarrow q) \vee (\sim p \rightarrow \sim q)$		
$\sim(p \rightarrow q) \vee (\sim q \rightarrow \sim p)$		

Exercise “Proper Logical Form” – 2

Connect the propositions (expressed in English) with their proper logical forms.

If Ann passes logic then she will either get married or she will find herself another boyfriend.	If Ann passes logic then she will need neither Bert’s help nor his love.	If Ann does not pass logic, then she will need to retake it, but if she passes then she will not need to retake it.	If Ann does not pass logic, then Bert will do all he can to help her; and, indeed, Ann will not pass logic.
$(\sim p \rightarrow q) \bullet (p \rightarrow \sim q)$	$p \rightarrow (q \vee r)$	$p \rightarrow (\sim q \bullet \sim r)$	$(\sim p \rightarrow q) \bullet \sim p$
If Chris buys ice-cream then he will buy neither cookies nor tea.	If Chris does not buy ice-cream then he will be miserable, but if he does buy ice-cream then he will not be miserable.	If Chris does not buy ice-cream then he will be miserable; in fact, Chris will not buy ice-cream.	If Chris buys ice-cream then he will eat it either immediately or within an hour.

If 10 is divisible by 5 then 10 is divisible by 1 and 2.	If 20 is divisible by 2 then if 40 is the result of multiplying 20 by 2 then 40 is divisible by 2 as well.	Either 10 is divisible by 2 or 10 is not divisible by 2 but by 1.	10 is divisible by 5 if and only if 10 is the product of 5 with some number.
$p \vee (\sim p \bullet q)$	$p \equiv q$	$p \rightarrow (q \bullet r)$	$p \rightarrow (q \rightarrow r)$
If you plant a seed then if you care for it properly, you will enjoy a beautiful plant.	If there is thunder, there is usually some heavy rain and hail.	Coffee is good if and only if it has half-&-half in it.	Chris will either get a dog, or he won’t get a dog but a cat.

2.2. Two Types of Logical Formulas: Propositions vs. Propositional Forms

I've already intimated that while propositional forms look to you like they are the same as propositions they in fact are not.

Every proposition says that something is the case and this is why it can be true or false. Propositions are thus the bearers of truth-values: they are either true or false.

Propositions state that something is the case. Every proposition is either true or false.

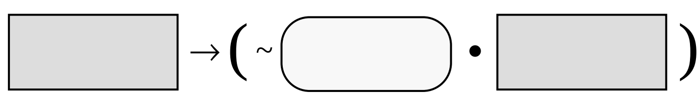
Propositional forms are not the sort of formulas that can be true or false! They are only abstract logical schemata. They don't say anything.

Propositional forms do not state anything. Propositional forms are neither true nor false.

We often get confused by this distinction because propositional forms are represented in terms of variables and we willy nilly underappreciate how an abstract notion the notion of a variable is. This is why whenever you are presented with a propositional form that looks thus:

$$p \rightarrow (\sim q \bullet p)$$

you should immediately convert the variables into boxes:

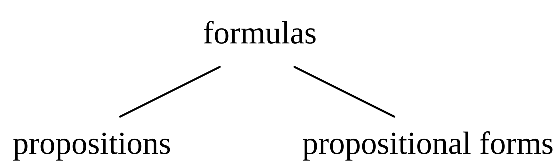


and you should be prepared to read out the propositional form thus:

If hmm-hmm then both not heh-heh and hmm-hmm.

I hope you agree that nothing true or false has been said here, for nothing at all has been said. We have only found a way of representing the logical structure of a statement. This is all that a propositional form does.

We will be using the term 'formula' to capture the "mother" category of the two categories of propositions and propositional forms:



3. Tautologies, Contradictions, Contingencies

Propositional forms can be: tautologous, contradictory or contingent.

3.1. Tautologies

It turns out (this is quite a remarkable fact if you think about it) that some propositional forms are such that no matter what statements you substitute for the propositional variables you will always get a true propositions as a result of the substitution. In other words, some propositional forms have only true substitution instances. Such propositional forms are called tautologies – and the substitution instances of those forms are logically true. Here is an example of such a form:

$$p \vee {\sim}p$$

When you substitute propositions for ‘p’, no matter whether true or false, you would always get a true proposition as a result. Think of a couple of propositions and insert them into the boxes to check in practice what is meant by being true in virtue of logical form.

Let’s do this systematically. First, let’s think of a proposition that is true for sure and insert it into the boxes:

Sharks are fish	or it is not the case that	sharks are fish	☑ true ☐ false

Think of at least two more true propositions and insert them into the propositional forms below and check that the resulting propositions (which are the substitution instances of the propositional form $p \vee {\sim}p$) are indeed true.

	or it is not the case that		☐ true ☐ false
	or it is not the case that		☐ true ☐ false

Now, let us think of a proposition that is most certainly false, like “Sharks are mammals”, and let’s insert such a proposition into the $p \vee {\sim}p$ propositional form. We will obtain as a result the following substitution instance of this form:

Sharks are mammals	or it is not the case that	sharks are mammals	☑ true ☐ false

Again the proposition is true. Think of two more examples of evidently false propositions and insert them into the propositional forms below:

	or it is not the case that		☐ true ☐ false
	or it is not the case that		☐ true ☐ false

Definition

Tautologies are such propositional forms all of whose substitution instances are true.

Note that we cannot speak of a propositional form being true (only propositions can be true). A propositional form is tautologous if all of its substitution instances are true.

3.2. Contradictions

There are other propositional forms with only false substitution instances – no matter what propositions you substitute for the propositional variables, you will always get a false proposition as a result. These propositional forms are called contradictions. Again recall the example of such a proposition:

$$p \bullet \sim p$$

Again, it does not matter whether you substitute a true proposition or a false proposition for *p*, the resulting proposition will be false.

First, let's insert a true proposition into the boxes (you should find two more to insert into the forms below and check that all three substitution instances are false):

Sharks are fish	and it is not the case that	sharks are fish	☐ true ☑ false
	and it is not the case that		☐ true ☐ false
	and it is not the case that		☐ true ☐ false

The resulting substitution instances are false. Now, let us insert false propositions into the $p \bullet \sim p$ propositional form:

Sharks are mammals	and it is not the case that	sharks are mammals	☐ true ☑ false
	and it is not the case that		☐ true ☐ false
	and it is not the case that		☐ true ☐ false

Again the resulting propositions are false.

Definition

Contradictions are propositional forms all of whose substitution instances are false.

3.3. Contingencies

Finally, there are propositional forms that do not determine the truth-value of their substitution instances – they are called contingencies. When a proposition that has the propositional form of a contingency is true, the proposition is said to be contingently true, when such a proposition is false, it is said to be contingently false. Consider the following propositional form:

$\sim p$

Depending on whether you substitute a false or a true proposition for *p*, the resulting proposition will be true or false, respectively.

It is not the case that	sharks are fish	☐ true ☑ false
It is not the case that		☐ true ☐ false
It is not the case that		☐ true ☐ false
It is not the case that	sharks are mammals	☐ true ☐ false
It is not the case that		☐ true ☐ false
It is not the case that		☐ true ☐ false

The logical form of the proposition $\sim p$ does not determine the truth-values of its substitution instances.

Definition

Contingencies are those propositional forms some of whose substitution instances are true and some of whose substitution instances are false

3.4. Properties of Propositional Forms vs. Properties of Propositions

Because propositions and propositional forms are different kinds of entities, they also differ in the properties they have. Propositions can be true or false, moreover they be true either logically or contingently or they can false either logically or contingently. Propositional forms on the other hand, cannot be either true or false. Propositional forms are tautologous, contradictory or contingent.

Moreover, there is a relationship between these properties of propositions and their logical forms, which is captured by the following table:

Propositional form	Propositions that are substitution instances of that form
Tautologous	Logically true
Contingent	Contingently true
	Contingently false
Contradictory	Logically false

3.5. Summary

In this section, we have distinguished three important types of propositional forms:

- tautologies – all of whose substitution instances are true,
- contradictions – all of whose substitution instances are false,
- contingencies – some of whose substitution instances are true and some of whose substitution instances are false.

We have also seen how the properties of propositions introduced in §1 relate to these properties of propositional forms:

- Logically true propositions are those propositions whose logical forms are tautologies.
- Logically false propositions are those propositions whose logical forms are contradictions.
- Contingently true propositions are propositions that are true and whose logical forms are contingencies.
- Contingently false propositions are propositions that are false and whose logical forms are contingencies.

4. The Origins of the Truth-Table Method (optional)

The concepts of a tautology, a contradiction, a contingency, on the one hand, as well as logical truth, contingent truth, contingent falsehood, logical falsehood, on the other, should be relatively clear to you by now. The problem is, however, that so far though we have an understanding of those concepts we have no method of finding out whether a given propositional form is a tautology, a contradiction or a contingency, and consequently no method for finding out whether a given proposition is logically or contingently true, or logically or contingently false.

4.1. The Problem

To see that this is a problem consider the definition of a tautology. A tautology is a propositional form *all of whose* substitution instances are true. To determine whether a given propositional form is a tautology we would have to find *all of* its substitution instances and determine that they are false. Now, depending on the resources of our language, 'all' might mean just 'incredibly many' but even 'infinitely many'. In either case, this definition is useless for all practical purposes. We just don't have the time to check all those instances. Consider the propositional form $p \vee {\sim}p$ again. We would have to check that it is true for *all* sentences of the English language:

> Mary is tired or Mary is not tired.
> Mary is bored or Mary is not bored.
> …
> Tim is tired or Tim is not tired.
> Tim is bored or Tim is not bored.
> …
> Sharks are mammals or sharks are not mammals
> Sharks are insect or sharks are not insects
> ….
> A tautology is a propositional form all of whose substitution instances are true or a tautology is not a propositional form all of whose substitution instances are true.
> ….
> It is very difficult to list all sentences of English or it is not very difficult to list all sentences of English.
> ….

It is very difficult to even begin listing all sentences of English (and we've began only listing very simple sentences, think about including more complicated ones), and there are linguists who believe that there is a potentially infinite number of English sentences.

4.2. The Solution

It is thus all the more remarkable that logicians *did* find a method for checking whether a propositional form is a tautology or not. How have they achieved it? Remember that logicians are lazy by nature. When presented with a propositional form like

$$p \vee \sim p$$

such a lazy logician has thought to himself:

> Why should I check the potentially infinite class of substitution instances of this propositional form when I can safely divide those substitution instances into two classes: those substitution instances that are formed by substituting a true proposition for *p*, and those substitution instances that are formed by substituting a false proposition for *p*.
>
> I should then consider what happens in each case. When I substitute a true proposition for *p*, I will get a true substitution instance of the propositional form because
>
> T ∨ ~T
> T ∨ F
> T
>
> But when I substitute a false proposition for *p*, I will also get a true substitution instance of the propositional form because
>
> F ∨ ~F
> F ∨ T
> T
>
> Since I can only either substitute a true or a false proposition for *p*, all the substitution instances of $p \vee \sim p$ will be true and I can conclude that $p \vee \sim p$ is a tautology.

I hope that you agree that this was pretty smart. In fact, this was the beginning of the truth-table method for checking whether a propositional form is a tautology, a contradiction or a contingency.

In case you are still wondering how the problem of checking the potentially infinite number of propositions is solved, you should reflect on the fact that the lazy logician divides class of substitution instances (which is potentially infinite) into (here:) two subclasses of substitution instances – however, each of the subclasses contains also a potentially infinite number of propositions.

5. Truth-Table Definitions of a Tautology, a Contradiction, a Contingency

If you have read the previous section you know how clever it was for logicians to invent the truth table method. But in any case, you now have the method and you need to learn it. We will be constructing truth tables for propositional forms to determine all the possible truth-values that the substitution instances of those forms can have. On this basis we will be able to determine whether the forms are tautologies, contradictions or contingencies.

A propositional form that is true in all rows of its truth table is a **tautology**.
A propositional form that is false in all rows of its truth table is a **contradiction**.
A propositional form that is true in at least one row of its truth table and false in at least one row of its truth table is a **contingency**.

5.1. Tautology – example

We have already said that $p \vee {\sim}p$ is a tautology. To show that this is so by means of a truth table we need to construct a truth table that will look thus:

	p	$p \vee {\sim}p$
1.	T	
2.	F	

In the propositional form $p \vee {\sim}p$ there is only one propositional variable p, which means that it can be substituted either by propositions that are true (considered in row 1) or by propositions that are false (considered in row 2). We thus need to calculate the truth-value of the complex propositions for each of these possibilities (rows). Let's do so step by step (you might want to do that above and check that you've done it correctly below).

We begin with row 1, first substituting the truth-values in that row for p in the form in question:

	p	$p \vee {\sim}p$
1.	T	T ∨ ~T
2.	F	

(The substitution step is one where most students make errors so always – especially with more complex truth tables, check that you have done it correctly).

Then you just proceed as you have learned in the previous unit. You calculate the truth-value of the substitution (or as we will also say: you calculate *the truth-value* *of the propositional form in that row*) except that, for convenience, we will be writing the steps next to each other, separated by columns:

	p	*p* ∨ ~*p*		
1.	T	T ∨ ~T	T ∨ F	T
2.	F			

We have thus shown that the propositional form *p* ∨ ~*p* is true in the first row of its truth table.

Let's show that the propositional form is also true in the second row of its truth table:

	p	*p* ∨ ~*p*		
1.	T	T ∨ ~T	T ∨ F	T
2.	F	F ∨ ~F	F ∨ T	T

The propositional form *p* ∨ ~*p* is a tautology because all of its substitution instances are true, which is shown by the truth table – the propositional form is true in all rows of its truth table.

As a point of terminology, you should remember that the highlighted part of the truth table is called the **truth-table base**. We will say more about how truth-table bases are constructed in §7.

5.2. Contradiction – example

Let's consider an example of a contradictory propositional form: *p* • ~*p*. Again there is only one propositional variable which can be substituted either by true or by false propositions. The truth table for *p* • ~*p* will again have only two rows:

	p	*p* • ~*p*		
1.	T			
2.	F			

You should try to fill in the truth table to see that the propositional form is false in all rows of the truth table.

	p	*p* • ~*p*		
1.	T	T • ~T	T • F	F
2.	F	F • ~F	F • T	F

5.3. Contingency – example

Let's consider an example of a contingency: $p \vee (p \bullet q)$. Now note, however, that there are two propositional variables in that propositional form, viz. *p* and *q*. We thus need to consider more possibilities – in fact exactly 4:

1. *p* and *q* may be substituted by true propositions
2. *p* may be substituted by true propositions while *q* is substituted by false propositions
3. *p* may be substituted by false propositions while *q* is substituted by true propositions
4. *p* and *q* may be substituted by false propositions

There are no other possibilities of substitutions. But this also means that we need to include those possibilities in our truth table. In fact, the truth-table base for a propositional form with two propositional variables looks thus:

	p	*q*	$p \vee (p \bullet q)$		
1.	T	T			
2.	T	F			
3.	F	T			
4.	F	F			

You should try to fill out the truth table on your own and check that you have done so correctly. There are two options for filling out a truth table. You might proceed row by row, as I suggested above, or you might proceed column by column, as we will proceed below; our first step will be substituting the truth-values under the variables:

	p	*q*	$p \vee (p \bullet q)$		
1.	T	T	T ∨ (T • T)		
2.	T	F	T ∨ (T • F)		
3.	F	T	F ∨ (F • T)		
4.	F	F	F ∨ (F • F)		

Check three times that you carried out the substitutions correctly! When you've done that you are ready to calculate; you might proceed by rows, or by columns, as is convenient for you:

	p	*q*	$p \vee (p \bullet q)$		
1.	T	T	T ∨ (T • T)	T ∨ T	**T**
2.	T	F	T ∨ (T • F)	T ∨ F	**T**
3.	F	T	F ∨ (F • T)	F ∨ F	**F**
4.	F	F	F ∨ (F • F)	F ∨ F	**F**

The propositional form $p \vee (p \bullet q)$ is a contingency – there are at least one row (in fact there are two such rows – rows 1 and 2) where it is true, and there is at least one row in fact there are two such rows – rows 3 and 4) where it is false.

Exercise Tautology—Contradiction—Contingency—1

Using the truth table method, determine whether a given propositional form is a tautology, a contradiction or a contingency. For each case, provide a *precise* justification.

(a)

p	$p \vee p$	
T	T ∨ T	T
F	F ∨ F	F

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row ___) and at least one row where it is false (viz. row ___).

(b)

p	$p \bullet p$	
T	T • T	T
F	F • F	F

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row ___) and at least one row where it is false (viz. row ___).

(c)

p	$p \rightarrow p$	
T	T → T	T
F	F → F	T

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row ___) and at least one row where it is false (viz. row ___).

(d)

p	$p \equiv p$	
T	T ≡ T	T
F	F ≡ F	T

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row ___) and at least one row where it is false (viz. row ___).

(e)

p	$p \rightarrow {\sim}p$		
T	T → ~T	T → F	F
F	F → ~F	F → T	T

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row ___) and at least one row where it is false (viz. row ___).

(f)

p	${\sim}p \rightarrow p$		
T	~T → T	F → T	T
F	~F → F	T → F	F

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row ___) and at least one row where it is false (viz. row ___).

(g)

p	$p \equiv {\sim}p$		
T			
F			

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row) and at least one row where it is false (viz. row).

(h)

p	${\sim}p \equiv p$		
T			
F			

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row) and at least one row where it is false (viz. row).

(i)

p	${\sim}(p \bullet {\sim}p)$			
T				
F				

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row) and at least one row where it is false (viz. row).

(j)

p	${\sim}(p \vee {\sim}p)$			
T				
F				

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row) and at least one row where it is false (viz. row).

(k)

p	${\sim}p$	
T		
F		

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row) and at least one row where it is false (viz. row).

(l)

p	p
T	
F	

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. row) and at least one row where it is false (viz. row).

Exercise Tautology—Contradiction—Contingency—2

Using the truth table method, determine whether a given propositional form is a tautology, a contradiction or a contingency. For each case, provide a *precise* justification.

(a)

p	q	$(p \bullet q) \rightarrow p$		
T	T			
T	F			
F	T			
F	F			

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. rows) and at least one row where it is false (viz. rows).

(b)

p	q	$p \rightarrow (p \bullet q)$		
T	T			
T	F			
F	T			
F	F			

The propositional form is a because:

(c)

p	q	$(p \vee q) \rightarrow p$		
T	T	(T • T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	T

The propositional form is:

☐ a tautology because it is true in all rows of the truth table
☐ a contradiction because it is false in all rows of the truth table
☐ a contingency because there is at least one row where it is true (viz. rows ______) and at least one row where it is false (viz. rows ______).

(d)

p	q	$p \rightarrow (p \vee q)$		
T	T	(T • T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	T

The propositional form is a ______________ because:

(e)

p	*q*	$(p \to q) \to (q \to p)$		
T	T	(T • T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	T

The propositional form is:

☐ a tautology because it is true in all rows of the truth table
☐ a contradiction because it is false in all rows of the truth table
☐ a contingency because there is at least one row where it is true (viz. rows ______) and at least one row where it is false (viz. rows ______).

(f)

p	*q*	$(p \to q) \to (\sim q \to \sim p)$			
T	T	(T • T) → T	(T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	(F) → F	T

The propositional form is a ______ because:

(g)

p	q	$p \equiv [p \vee (p \rightarrow q)]$			
T	T	(T • T) → T	(T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	(F) → F	T

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. rows ______) and at least one row where it is false (viz. rows ______).

(h)

p	q	$[p \bullet (p \rightarrow q)] \rightarrow q$			
T	T	(T • T) → T	(T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	(F) → F	T

The propositional form is a ______________ because:

(i)

p	q	$[p \bullet (p \to q)] \to \sim q$			
T	T	(T • T) → T	(T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	(F) → F	T

The propositional form is:

☐ a tautology because it is true in all rows of the truth table

☐ a contradiction because it is false in all rows of the truth table

☐ a contingency because there is at least one row where it is true (viz. rows ______) and at least one row where it is false (viz. rows ______).

(j)

p	q	$\sim\{[p \bullet (p \to q)] \to q\}$				
T	T	(T • T) → T	(T) → T	(T) → T	T	T
T	F	(T • F) → T	(F) → T	(F) → T	T	T
F	T	(F • T) → F	(F) → F	(F) → F	T	T
F	F	(F • F) → F	(F) → F	(F) → F	T	T

The propositional form is a ______________ because:

Exercise Tautology—Contradiction—Contingency—3

Using the truth table method, determine whether a given propositional form is a tautology, a contradiction or a contingency. For each case, provide a *precise* justification.

(a)

p	*q*	~(*p* • *q*)
T	T	
T	F	
F	T	
F	F	

The propositional form is:

☐ a tautology because it is true in all rows of the truth table

☐ a contradiction because it is false in all rows of the truth table

☐ a contingency because there is at least one row where it is true (viz. rows) and at least one row where it is false (viz. rows).

(b)

p	*q*	~*p* ∨ ~*q*
T	T	
T	F	
F	T	
F	F	

The propositional form is a because:

(c)

p	q	$(p \vee q) \rightarrow \sim p$
T	T	(T • T) → T (T) → T (T) → T T
T	F	(T • F) → T (F) → T (F) → T T
F	T	(F • T) → F (F) → F (F) → F T
F	F	(F • F) → F (F) → F (F) → F T

The propositional form is:

☐ a tautology because it is true in all rows of the truth table
☐ a contradiction because it is false in all rows of the truth table
☐ a contingency because there is at least one row where it is true (viz. rows) and at least one row where it is false (viz. rows).

(d)

p	q	$\sim p \rightarrow (p \vee q)$
T	T	(T • T) → T (T) → T (T) → T T
T	F	(T • F) → T (F) → T (F) → T T
F	T	(F • T) → F (F) → F (F) → F T
F	F	(F • F) → F (F) → F (F) → F T

The propositional form is a because:

(e)

p	*q*	$(p \to q) \equiv (\sim p \lor q)$			
T	T	(T • T) → T	(T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	(F) → F	T

The propositional form is:

☐ a tautology because it is true in all rows of the truth table
☐ a contradiction because it is false in all rows of the truth table
☐ a contingency because there is at least one row where it is true (viz. rows ______) and at least one row where it is false (viz. rows ______).

(f)

p	*q*	$\sim(p \to q) \equiv (\sim p \lor q)$			
T	T	(T • T) → T	(T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	(F) → F	T

The propositional form is a ______________________ because:

(g)

p	q	$\sim(p \to q) \equiv (p \bullet \sim q)$			
T	T	(T • T) → T	(T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	(F) → F	T

The propositional form is:

☐ a tautology because it is true in all rows of the truth table
☐ a contradiction because it is false in all rows of the truth table
☐ a contingency because there is at least one row where it is true (viz. rows ______) and at least one row where it is false (viz. rows ______).

(h)

p	q	$(p \to q) \to [p \vee (p \to q)]$			
T	T	(T • T) → T	(T) → T	(T) → T	T
T	F	(T • F) → T	(F) → T	(F) → T	T
F	T	(F • T) → F	(F) → F	(F) → F	T
F	F	(F • F) → F	(F) → F	(F) → F	T

The propositional form is a ______________ because:

(i)

p	*q*	$[p \vee (p \to q)] \to (p \to q)$
T	T	
T	F	
F	T	
F	F	

The propositional form is:

- ☐ a tautology because it is true in all rows of the truth table
- ☐ a contradiction because it is false in all rows of the truth table
- ☐ a contingency because there is at least one row where it is true (viz. rows ______) and at least one row where it is false (viz. rows ______).

(j)

p	*q*	*r*	$[(p \to q) \bullet (q \to r)] \to (p \to r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

The propositional form is a ______________ because:

6. Determining the Properties of Propositions

6.1. Reminder: Properties of Propositions vs. Properties of Propositional Forms

We began this section with distinguishing two types of truth – logical truth and contingent truth and two types of falsehood – logical falsehood and contingent falsehood. We have, moreover, claimed that the logical forms of propositions that are contingently true or contingently false are contingencies, while the logical forms of logically true propositions are tautologies and the logical forms of logically false propositions are contradictions. This is summarized in the following table, which will be useful to see once again.

Propositional form	Propositions that are substitution instances of that form
Tautologous	Logically true
Contingent	Contingently true
	Contingently false
Contradictory	Logically false

In this section, we will see how to determine whether a certain English statement is true – contingently or logically, or false – contingently or logically. Essentially, we will first determine whether it is true or false and then determine whether its propositional form is a tautology, a contingency or a contradiction.

6.2. Example 1 (contingent falsehood)

Is the following statement contingently or logically true, or contingently or logically false?

(1) Some roses smell nice while some roses smell like dirty socks.

The first thing we do is to symbolize the statement:

N: Some roses smell nice
D: Some roses smell like dirty socks

[1] N • D

The next thing we need to do is to determine what the truth-values of the simple propositions are (here N is true, while D is false), and to determine what the resulting truth-value of the proposition (1) is:

T • F
F

Proposition (1) is false. The question remains whether it is logically or contingently false. To determine that we need to identify the proper logical form of the proposition:

{1} $p \bullet q$

and to construct a truth table for the propositional form:

p	q	$p \bullet q$	
T	T	T • T	T
T	F	T • F	F
F	T	F • T	F
F	F	F • F	F

The propositional form $p \bullet q$ is a contingency since there is at least one row where the form is true (row 1) and at least one row where it is false (rows 2, 3, 4).

Since proposition (1) is false and its proper logical form is a contingency, we can conclude that proposition (1) is contingently false.

6.3. Example 2 (contingent truth)

Is the following statement contingently or logically true, or contingently or logically false?

(2) Cows are neither insects nor reptiles.

The first thing we do is to symbolize the statement:

I: Cows are insects
R: Cows are reptiles

[2] ~I • ~R

(There is also the alternative symbolization available ~(I ∨ R) but let's stick with [2].) The next thing we need to do is to determine what the truth-values of the simple propositions are (here I is false, and R is false too), and to determine what the resulting truth-value of the proposition (2) is:

~F • ~F
T • T
T

Proposition (2) is thus true (which we also intuitively feel is the case). The question remains whether it is logically or contingently true. To determine that we need to identify the proper logical form of the proposition:

{2} $\sim p \bullet \sim q$

and to construct a truth table for the propositional form:

p	q	$\sim p \bullet \sim q$		
T	T	~T • ~T	F • F	F
T	F	~T • ~F	F • T	F
F	T	~F • ~T	T • F	F
F	F	~F • ~F	T • T	T

The propositional form $\sim p \bullet \sim q$ is a contingency since there is at least one row where the form is true (row 4) and at least one row where it is false (rows 1, 2, 3).

Since proposition (2) is true and its proper logical form is a contingency, we can conclude that proposition (2) is contingently true.

6.4. Example 3 (logical truth)

Is the following statement contingently or logically true, or contingently or logically false?

(3) If logic is not both useful and interesting then logic is either not useful or not interesting.

The first thing we do is to symbolize the statement:

I: Logic is interesting
U: Logic is useful

[3] $\sim(U \bullet I) \rightarrow (\sim U \vee \sim I)$

The next thing we need to do is to determine what the truth-values of the simple propositions are (here I is true, and U is true, too – actually, as you will see, it does not matter what truth-values you'd assign to the simple propositions in this case), and to determine what the resulting truth-value of the proposition (3) is:

$$\sim(T \bullet T) \rightarrow (\sim T \vee \sim T)$$
$$\sim(T) \rightarrow (F \vee F)$$
$$F \rightarrow F$$
$$T$$

Proposition (3) is thus true. The question remains whether it is logically or contingently true. To determine that we need to identify the proper logical form of the proposition:

{3} $\sim(p \bullet q) \rightarrow (\sim p \vee \sim q)$

and to construct a truth table for the propositional form:

p	q	$\sim(p \bullet q) \rightarrow (\sim p \vee \sim q)$			
T	T	~(T • T) → (~T ∨ ~T)	~(T) → (F ∨ F)	F → (F)	T
T	F	~(T • F) → (~T ∨ ~F)	~(F) → (F ∨ T)	T → (T)	T
F	T	~(F • T) → (~F ∨ ~T)	~(F) → (T ∨ F)	T → (T)	T
F	F	~(F • F) → (~F ∨ ~F)	~(F) → (T ∨ T)	T → (T)	T

The propositional form $\sim(p \bullet q) \rightarrow (\sim p \vee \sim q)$ is a tautology since it is true in all rows of its truth table (this is also why it did not matter much in this case whether you agreed with me that the simple propositions are true).

Since the proper logical form of proposition (3) is a tautology (which means that proposition (3) is true), we can conclude that proposition (3) is logically true.

6.5. Example 4 (logical falsehood)

Is the following statement contingently or logically true, or contingently or logically false?

(4) Love is eternal if and only if love is not eternal.

The first thing we do is to symbolize the statement:

L: love is eternal

[4] L ≡ ~L

The next thing we need to do is to determine what the truth-values of the simple propositions are (suppose that we accept L as true – though, as you will see, it does not matter what truth-values we assign to the simple proposition in this case), and to determine what the resulting truth-value of the proposition (4) is:

T ≡ ~T
T ≡ F
F

Proposition (4) is thus false. The question remains whether it is logically or contingently false. To determine that we need to identify the proper logical form of the proposition:

{4} $p \equiv \sim p$

and to construct a truth table for the propositional form:

p	$p \equiv \sim p$		
T	T ≡ ~T	T ≡ F	**F**
F	F ≡ ~F	F ≡ T	**F**

The propositional form $p \equiv \sim p$ is a contradiction since it is false in all rows of its truth table (this is also why it did not matter much in this case whether you agreed with me that the simple propositions is true).

Since the proper logical form of proposition (4) is a contradiction (which means that proposition (4) is false), we can conclude that proposition (4) is logically false.

Exercise "Logical and Contingent Truth and Falsehood"

In the following cases, determine whether a proposition provided is true or false, and whether the character of its truth of falsehood is logical or contingent. Provide (1) a symbolization, (assignment of truth-values to simple proposition(s) is provided with the symbolization legend), (2) a calculation of the truth-value of the proposition, (3) the proper logical form of the proposition, (4) the truth table for that form, (5) a clear statement whether the propositional form is a tautology, a contingency or a contradiction with an explanation, (6) a clear statement whether the proposition is logically/contingently true/false with a full explanation. I provide a symbolization legend and an assignment of truth-values below:

C: Cancun is the capital of the Mexico – false
E: El Dorado is the capital of the Mexico – false
M: Mexico City is the capital of Mexico – true
U: Washington, D.C. is the capital of the U.S.A. – true

(a)
If Washington, D.C. is the capital of the U.S.A. then Mexico City is the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(b)

If Washington, D.C. is the capital of the U.S.A. then Washington, D.C. is the capital of the U.S.A.

Symbolization:

Truth-value calculation:

Proper logical form:

Truth-table:

The propositional form is:

because:

The proposition is:

because:

(c)

If Cancun is the capital of Mexico then Cancun is the capital of Mexico.

Symbolization:

Truth-value calculation:

Proper logical form:

Truth-table:

The propositional form is:

because:

The proposition is:

because:

(d)

Either Cancun is the capital of Mexico or Cancun is the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(e)

Either Cancun is the capital of Mexico or Cancun is not the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(f)

If either Cancun or Mexico City is the capital of Mexico then Cancun is the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(g)

If Cancun is the capital of Mexico then either Cancun or El Dorado is the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(h)
If Mexico City is the capital of Mexico then either Cancun or Mexico City is the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(i)
Neither Cancun nor El Dorado are the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(j)
Neither Cancun nor Mexico City are the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(k)

Either Cancun or Mexico City are the capital of Mexico, but not both.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(l)
Cancun is the capital of Mexico, however, neither El Dorado nor Cancun is the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(m)
Cancun is not the capital of Mexico while El Dorado is the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(n)
Cancun is the capital of Mexico provided that El Dorado is not the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

(o)

Either Cancun is the capital of Mexico just in case Cancun is not the capital of Mexico or El Dorado is the capital of Mexico just in case El Dorado is not the capital of Mexico.

Symbolization:	
Truth-value calculation:	
Proper logical form:	
Truth-table:	
The propositional form is:	
because:	
The proposition is:	
because:	

7. How to Construct a Truth-Table Base

So far we have been dealing with propositional forms that contain one or two variables. For such propositional forms it is best to just memorize their truth table bases. (Note that the order of the rows is important for conventional reasons – you should stick to it.) The truth-table base for a propositional form with one variable will be:

p
T
F

and for two variables:

p	*q*
T	T
T	F
F	T
F	F

But when it comes to truth table bases for more than two variables, it is better to know the systematic way of constructing them. The truth-table base for a propositional form with three variables will have 8 rows and will look thus:

p	*q*	*r*
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

In general, a truth-table base for a propositional form with *n* variables will have 2^n (2 to the power of *n*, i.e. you need to carry the multiplication $2 \cdot \ldots \cdot 2$, where there are *n* 2s to multiply) rows. Thus:

- when there is 1 variable, the truth-table base has 2 rows (because $2^1 = 2$)
- when there are 2 variables, the truth-table base has 4 rows (because $2^2 = 4$, i.e. $2 \cdot 2 = 4$)
- when there are 3 variables, the truth-table base has 8 rows (because $2^3 = 8$, i.e. $2 \cdot 2 \cdot 2 = 8$)
- when there are 4 variables, the truth-table base has 16 rows (because $2^4 = 16$, i.e. $2 \cdot 2 \cdot 2 \cdot 2 = 16$)
- when there are 5 variables, the truth-table base has 32 rows (because $2^5 = 32$, i.e. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$)
- and so on

The Algorithm for Constructing a Truth-Table Base

I provide here an algorithm for constructing a truth-table base for an arbitrarily large number of variables. Let us suppose that there are four variables. The first thing to do is to list them in an alphabetical order

p	*q*	*r*	*s*

Then beginning with the rightmost variable (here: *s*) construct a truth table for *s*.

p	*q*	*r*	*s*
			T
			F

Copy the truth-table for *s* beneath itself,

p	*q*	*r*	*s*
			T
			F
			T
			F

and to the left (here: under *r*) type T's next to the original truth table, and F's next to the copied truth table

p	*q*	*r*	*s*
		T	T
		T	F
		F	T
		F	F

When you look at the truth table under *r* and *s*, this is the regular 2-variable truth table. Indeed the next step is to copy the whole thing underneath itself once again, and in the column to the left (here: under *q*) write T's next to the original truth table and F's next to the copied truth table:

p	*q*	*r*	*s*
	T	T	T
	T	T	F
	T	F	T
	T	F	F
	F	T	T
	F	T	F
	F	F	T
	F	F	F

The truth table under *q*, *r* and *s* is a 3-variable truth table. The next step is to copy the whole 3-variable truth table underneath itself once more, and in the column to the left (here: under *p*) write T's next to the original truth table and F's next to the copied truth table:

p	*q*	*r*	*s*
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F

The truth table for 4-variables is thus completed. But obviously the same procedure could be continued if there were more than 4 more variables.

PowerPoint Presentation

I provide a simple MS PowerPoint show to show two different ways of constructing the truth table bases. Please note that in order to run the presentation, you need to have a MS PowerPoint on your computer. (If your computer does not have MS PowerPoint, you can probably find such a computer easily – in a computer center or in a library.)

However, the file, which contains the show, called

`TruthTableBases.pps`

can be run from outside the PowerPoint program. Double-click on the file in the Windows Explorer or select *Run* and type the file name (including its location). It is preferable to download the file rather than view it on-line because there are some kinds of strange hang-ups to the presentation when you try to view it immediately on-line.

What You Need to Know and Do

- You need to know the difference between propositions and propositional forms
- You need to be able to tell whether a formula is a proposition or a propositional form
- You need to know that propositions and propositional forms have different properties
- You need to know how the properties of propositional forms (tautologousness, contradictoriness, contingency) relate to the properties of propositions (logical truth, logical falsehood, contingent truth, contingent falsehood)
- You need to know the definition of tautologies, contradictions and contingencies as their truth-table operationalization.
- You need to be able to construct the base of the truth table for up to four variables (in the order in which we have constructed them).
- You need to know how to choose the truth-table base for a proposition depending on the number of variables in it.
- You need to be able to use the truth table method to tell whether a propositional form is a tautology, a contradiction or a contingency.
- You need to be to use the methods here introduced to decide whether a proposition is logically or contingently true or false.