

Solutions to Workbook Exercises

Unit 5:

Calculating Truth-Values of Complex Propositions (Part II)

Exercise Truth-Values (B) – 1

$$\begin{aligned}
 1. & \quad \underbrace{\sim T}_{F} \vee \underbrace{(\sim T \rightarrow F)}_{T} \\
 & \quad F \vee T \\
 & \quad T
 \end{aligned}$$

$$\begin{aligned}
 2. & \quad \underbrace{\sim(T \equiv T)}_{F} \vee \underbrace{\sim(F \equiv F)}_{F} \\
 & \quad \sim T \vee \sim T \\
 & \quad F \vee F \\
 & \quad F
 \end{aligned}$$

$$\begin{aligned}
 3. & \quad \underbrace{\sim(T \rightarrow T)}_{F} \vee \underbrace{(\sim F \rightarrow F)}_{F} \\
 & \quad \sim T \vee (T \rightarrow F) \\
 & \quad F \vee F \\
 & \quad F
 \end{aligned}$$

$$\begin{aligned}
 4. & \quad \underbrace{(\sim T \cdot \sim T)}_{F} \vee \underbrace{\sim(F \equiv F)}_{F} \\
 & \quad (F \cdot F) \vee \sim T \\
 & \quad F \vee F \\
 & \quad F
 \end{aligned}$$

$$\begin{aligned}
 5. & \quad \underbrace{\sim T}_{F} \equiv \underbrace{(\sim T \vee (\sim F \rightarrow F))}_{T} \\
 & \quad F \equiv (F \vee (T \rightarrow F)) \\
 & \quad F \equiv (F \vee F) \\
 & \quad F \equiv F \\
 & \quad T
 \end{aligned}$$

$$\begin{aligned}
 6. & \quad \underbrace{(T \equiv T)}_{T} \cdot \underbrace{(\sim T \equiv (\sim T \rightarrow F))}_{F} \\
 & \quad T \cdot (F \equiv (F \rightarrow F)) \\
 & \quad T \cdot (F \equiv T) \\
 & \quad T \cdot F \\
 & \quad F
 \end{aligned}$$

$$\begin{aligned}
 7. & \quad \underbrace{\sim(F \equiv F)}_{F} \rightarrow \underbrace{\sim(F \vee (T \rightarrow F))}_{F} \\
 & \quad \sim T \rightarrow \sim(F \vee F) \\
 & \quad F \rightarrow \sim F \\
 & \quad F \rightarrow T \\
 & \quad T
 \end{aligned}$$

$$\begin{aligned}
 8. & \quad \underbrace{\sim[(F \equiv F) \vee \sim(F \vee F)]}_{F} \equiv \underbrace{(T \cdot F)}_{F} \\
 & \quad \sim[T \vee \sim F] \equiv F \\
 & \quad \sim[T \vee T] \equiv F \\
 & \quad \sim T \equiv F \\
 & \quad F \equiv F \\
 & \quad T
 \end{aligned}$$

Exercise Truth-Values (B) – 2

$$\begin{array}{l} 1. \sim T \vee T \\ \quad F \vee T \\ \quad \quad T \end{array}$$

$$\begin{array}{l} 2. \sim (T \vee T) \\ \quad \sim (T) \\ \quad \quad F \end{array}$$

$$\begin{array}{l} 3. \sim T \vee \sim T \\ \quad F \vee F \\ \quad \quad F \end{array}$$

$$\begin{array}{l} 4. \sim (T \rightarrow T) \\ \quad \sim (T) \\ \quad \quad F \end{array}$$

$$\begin{array}{l} 5. \sim T \rightarrow T \\ \quad F \rightarrow T \\ \quad \quad T \end{array}$$

$$\begin{array}{l} 6. \sim T \rightarrow \sim T \\ \quad F \rightarrow F \\ \quad \quad T \end{array}$$

$$\begin{array}{l} 7. \sim T \vee (\sim T \rightarrow F) \\ \quad F \vee (F \rightarrow F) \\ \quad \quad F \vee (T) \\ \quad \quad \quad T \end{array}$$

$$\begin{array}{l} 8. \sim T \rightarrow \sim (T \equiv F) \\ \quad F \rightarrow \sim (F) \\ \quad \quad F \rightarrow T \\ \quad \quad \quad T \end{array}$$

$$\begin{array}{l} 9. (\sim T \rightarrow \sim T) \vee F \\ \quad (F \rightarrow F) \vee F \\ \quad \quad (T) \vee F \\ \quad \quad \quad T \end{array}$$

$$\begin{array}{l} 10. \sim (T \bullet T) \rightarrow F \\ \quad \quad \sim (T) \rightarrow F \\ \quad \quad \quad F \rightarrow F \\ \quad \quad \quad \quad T \end{array}$$

$$\begin{array}{l} 11. (\sim F \vee \sim F) \bullet (\sim T \vee \sim T) \\ \quad (T \vee T) \bullet (F \vee F) \\ \quad \quad (T) \bullet (F) \\ \quad \quad \quad F \end{array}$$

$$\begin{array}{l} 12. \sim (F \vee F) \bullet \sim (F \equiv T) \\ \quad \quad \sim (F) \bullet \sim (F) \\ \quad \quad \quad T \bullet T \\ \quad \quad \quad \quad T \end{array}$$

$$\begin{array}{l} 13. \sim F \rightarrow [\sim F \bullet (\sim T \vee \sim T)] \\ \quad T \rightarrow [T \bullet (F \vee F)] \\ \quad T \rightarrow [T \bullet (F)] \\ \quad T \rightarrow [F] \\ \quad \quad F \end{array}$$

$$\begin{array}{l} 14. \sim F \bullet \sim [F \vee (T \equiv T)] \\ \quad T \bullet \sim [F \vee (T)] \\ \quad T \bullet \sim [T] \\ \quad T \bullet F \\ \quad \quad F \end{array}$$

$$\begin{array}{l} 15. [\sim T \equiv (\sim F \bullet \sim T)] \rightarrow \sim T \\ \quad [F \equiv (T \bullet F)] \rightarrow F \\ \quad [F \equiv (F)] \rightarrow F \\ \quad \quad [T] \rightarrow F \\ \quad \quad \quad F \end{array}$$

$$\begin{array}{l} 16. \sim T \bullet \sim [F \vee \sim (T \vee F)] \\ \quad F \bullet \sim [F \vee \sim (T)] \\ \quad F \bullet \sim [F \vee F] \\ \quad F \bullet \sim [F] \\ \quad F \bullet T \\ \quad \quad F \end{array}$$

Exercise Truth-Values (B) – 3

$$1. T \vee \sim(F \rightarrow \sim F)$$

$$T \vee \sim(F \rightarrow T)$$

$$T \vee \sim T$$

$$T \vee F$$

$$T$$

$$2. \sim F \bullet \sim(\sim F \vee \sim F)$$

$$T \bullet \sim(T \vee T)$$

$$T \bullet \sim T$$

$$T \bullet F$$

$$F$$

$$3. \sim[F \equiv \sim(F \vee T)]$$

$$\sim[F \equiv \sim T]$$

$$\sim[F \equiv F]$$

$$\sim T$$

$$F$$

$$4. \sim[T \vee \sim(F \rightarrow \sim F)]$$

$$\sim[T \vee \sim(F \rightarrow T)]$$

$$\sim[T \vee \sim T]$$

$$\sim[T \vee F]$$

$$\sim T$$

$$F$$

$$5. \sim T \bullet \sim(T \vee (\sim F \rightarrow F))$$

$$F \bullet \sim(T \vee (T \rightarrow F))$$

$$F \bullet \sim(T \vee F)$$

$$F \bullet \sim T$$

$$F \bullet F$$

$$F$$

$$6. \sim(T \vee \sim F) \equiv (\sim T \vee \sim(T \rightarrow F))$$

$$\sim(T \vee T) \equiv (F \vee \sim F)$$

$$\sim T \equiv (F \vee T)$$

$$F \equiv T$$

$$F$$

Exercise Truth-Values (B) – 4

$$\begin{aligned}
 1. \quad & (\sim F \bullet \sim T) \equiv \sim (T \equiv \sim F) \\
 & (T \bullet F) \equiv \sim (T \equiv T) \\
 & (F) \equiv \sim (T) \\
 & F \equiv F \\
 & T
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \sim (F \vee \sim F) \rightarrow \sim (F \vee \sim T) \\
 & \sim (F \vee T) \rightarrow \sim (F \vee F) \\
 & \sim (T) \rightarrow \sim (F) \\
 & F \rightarrow T \\
 & T
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \sim (\sim F \bullet \sim F) \rightarrow \sim (F \rightarrow \sim T) \\
 & \sim (T \bullet T) \rightarrow \sim (F \rightarrow F) \\
 & \sim (T) \rightarrow \sim (T) \\
 & F \rightarrow F \\
 & T
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \sim [(\sim F \equiv \sim F) \bullet \sim F] \\
 & \sim [(T \equiv T) \bullet T] \\
 & \sim [(T) \bullet T] \\
 & \sim [T] \\
 & F
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \sim [(\sim F \bullet \sim F) \rightarrow \sim (F \rightarrow \sim T)] \\
 & \sim [(T \bullet T) \rightarrow \sim (F \rightarrow F)] \\
 & \sim [(T) \rightarrow \sim (T)] \\
 & \sim [T \rightarrow F] \\
 & \sim [F] \\
 & T
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \sim [(\sim T \equiv \sim F) \bullet \sim F] \rightarrow \sim T \\
 & \sim [(F \equiv T) \bullet T] \rightarrow F \\
 & \sim [(F) \bullet T] \rightarrow F \\
 & \sim [F] \rightarrow F \\
 & T \rightarrow F \\
 & F
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \sim [\sim (T \equiv F) \bullet \sim F] \vee \sim T \\
 & \sim [\sim (F) \bullet T] \vee F \\
 & \sim [T \bullet T] \vee F \\
 & \sim [T] \vee F \\
 & F \vee F \\
 & F
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \sim [\sim (\sim T \vee \sim F) \bullet \sim F] \rightarrow \sim T \\
 & \sim [\sim (F \vee T) \bullet T] \rightarrow F \\
 & \sim [\sim (T) \bullet T] \rightarrow F \\
 & \sim [F \bullet T] \rightarrow F \\
 & \sim [F] \rightarrow F \\
 & T \rightarrow F \\
 & F
 \end{aligned}$$

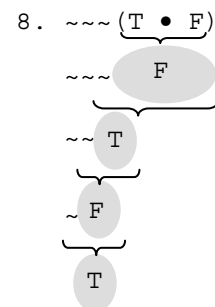
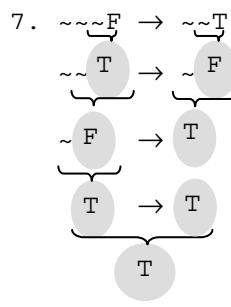
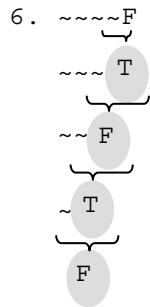
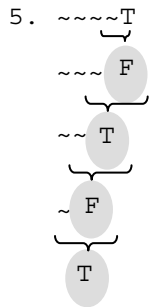
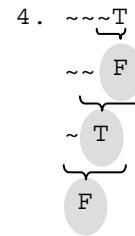
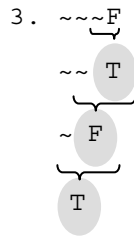
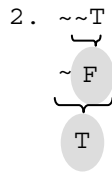
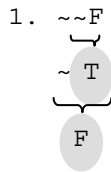
$$\begin{aligned}
 9. \quad & \sim T \vee \sim [\sim F \bullet \sim (T \rightarrow \sim F)] \\
 & F \vee \sim [T \bullet \sim (T \rightarrow T)] \\
 & F \vee \sim [T \bullet \sim (T)] \\
 & F \vee \sim [T \bullet F] \\
 & F \vee \sim [F] \\
 & F \vee T \\
 & T
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \sim \{T \vee \sim [\sim F \bullet \sim (T \rightarrow \sim F)]\} \\
 & \sim \{T \vee \sim [T \bullet \sim (T \rightarrow T)]\} \\
 & \sim \{T \vee \sim [T \bullet \sim (T)]\} \\
 & \sim \{T \vee \sim [T \bullet F]\} \\
 & \sim \{T \vee \sim [F]\} \\
 & \sim \{T \vee T\} \\
 & \sim \{T\} \\
 & F
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \sim \{\sim T \equiv \sim [\sim T \vee \sim (\sim F \bullet \sim F)]\} \\
 & \sim \{F \equiv \sim [F \vee \sim (T \bullet T)]\} \\
 & \sim \{F \equiv \sim [F \vee \sim (T)]\} \\
 & \sim \{F \equiv \sim [F \vee F]\} \\
 & \sim \{F \equiv \sim [F]\} \\
 & \sim \{F \equiv T\} \\
 & \sim \{F\} \\
 & T
 \end{aligned}$$

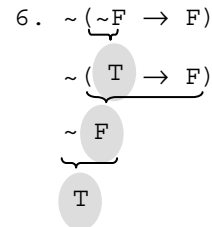
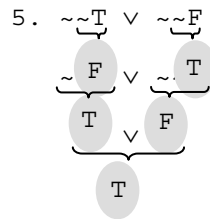
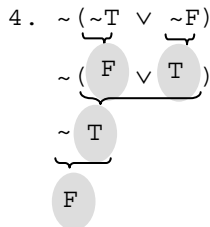
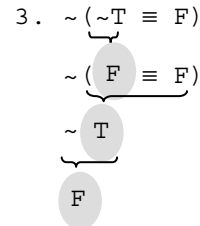
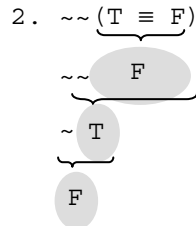
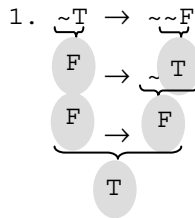
$$\begin{aligned}
 12. \quad & \sim (\sim T \vee \sim F) \equiv \sim [F \equiv \sim (F \bullet \sim F)] \\
 & \sim (F \vee T) \equiv \sim [F \equiv \sim (F \bullet T)] \\
 & \sim (T) \equiv \sim [F \equiv \sim (F)] \\
 & F \equiv \sim [F \equiv T] \\
 & F \equiv \sim [F] \\
 & F \equiv T \\
 & F
 \end{aligned}$$

Exercise Truth-Values Multiple-Negations



Exercise Truth-Values (B) – 5

Complete the following calculations. Check that your results match the results in the *Solutions* before proceeding.



Exercise Truth-Values (B) – 6

In this exercise, you will be provided with the schemata for carrying out the calculations. Make sure that you understand why you proceed in this order. In the next exercises, you will be “on your own.” Check that your results match the results in the Solutions before proceeding.

$$\begin{array}{l}
 1. \quad \sim\sim T \bullet \sim(\sim T \bullet F) \\
 \underbrace{\sim\sim T}_{F} \bullet \underbrace{\sim(\sim T \bullet F)}_{\sim(F \bullet F)} \\
 T \bullet \sim F \\
 T \bullet T \\
 \underbrace{\quad}_{T}
 \end{array}$$

$$\begin{array}{l}
 2. \quad \sim\sim T \bullet (\sim\sim T \bullet F) \\
 \underbrace{\sim\sim T}_{F} \bullet \underbrace{(\sim\sim T \bullet F)}_{\sim(T \bullet F)} \\
 T \bullet (T \bullet F) \\
 T \bullet F \\
 \underbrace{\quad}_{F}
 \end{array}$$

$$\begin{array}{l}
 3. \quad \sim[\sim T \equiv \sim(\sim F \bullet \sim\sim T)] \\
 \sim[F \equiv \sim(T \bullet \sim F)] \\
 \sim[F \equiv \sim(T \bullet T)] \\
 \sim[F \equiv \sim T] \\
 \sim[F \equiv F] \\
 \sim T \\
 \underbrace{\quad}_{F}
 \end{array}$$

$$\begin{array}{l}
 4. \quad \sim\sim[T \equiv \sim(\sim F \bullet \sim\sim T)] \\
 \sim\sim[T \equiv \sim(T \bullet \sim F)] \\
 \sim\sim[T \equiv \sim(T \bullet T)] \\
 \sim\sim[T \equiv \sim T] \\
 \sim\sim[T \equiv F] \\
 \sim\sim F \\
 \sim T \\
 \underbrace{\quad}_{F}
 \end{array}$$

$$\begin{array}{l}
 5. \quad \sim[T \rightarrow \sim(T \equiv \sim\sim\sim T)] \\
 \sim[T \rightarrow \sim(T \equiv \sim\sim F)] \\
 \sim[T \rightarrow \sim(T \equiv \sim T)] \\
 \sim[T \rightarrow \sim(T \equiv F)] \\
 \sim[T \rightarrow \sim F] \\
 \sim[T \rightarrow T] \\
 \sim T \\
 \underbrace{\quad}_{F}
 \end{array}$$

$$\begin{array}{l}
 6. \quad \sim[\sim(\sim F \rightarrow \sim\sim\sim T) \bullet \sim\sim\sim F] \\
 \sim[\sim(T \rightarrow \sim\sim F) \bullet \sim\sim T] \\
 \sim[\sim(T \rightarrow \sim T) \bullet \sim F] \\
 \sim[\sim(T \rightarrow F) \bullet T] \\
 \sim[\sim F \bullet T] \\
 \sim[T \bullet T] \\
 \sim T \\
 \underbrace{\quad}_{F}
 \end{array}$$

Exercise Truth-Values (B) – 7

$$\begin{aligned}
 1. \quad & \sim\sim T \vee \sim\sim F \\
 & \sim F \vee \sim T \\
 & T \vee F \\
 & T
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \sim(\sim T \vee \sim F) \\
 & \sim(F \vee T) \\
 & \sim(T) \\
 & F
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \sim\sim\sim F \rightarrow \sim\sim\sim T \\
 & \sim\sim T \rightarrow \sim\sim\sim F \\
 & \sim F \rightarrow \sim\sim T \\
 & T \rightarrow \sim F \\
 & T \rightarrow T \\
 & T
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \sim\sim\sim(F \rightarrow \sim T) \\
 & \sim\sim\sim(F \rightarrow F) \\
 & \sim\sim\sim(T) \\
 & \sim\sim F \\
 & \sim T \\
 & F
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \sim\sim(T \vee \sim T) \\
 & \sim\sim(T \vee F) \\
 & \sim\sim(T) \\
 & \sim F \\
 & T
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \sim\sim(\sim\sim T \rightarrow \sim T) \\
 & \sim\sim(\sim F \rightarrow F) \\
 & \sim\sim(T \rightarrow F) \\
 & \sim\sim(F) \\
 & \sim T \\
 & F
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \sim(T \bullet \sim T) \rightarrow \sim(\sim F \vee \sim F) \\
 & \sim(T \bullet F) \rightarrow \sim(T \vee T) \\
 & \sim(F) \rightarrow \sim(T) \\
 & T \rightarrow F \\
 & F
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \sim[(\sim T \vee \sim T) \bullet \sim(F \vee \sim F)] \\
 & \sim[(F \vee F) \bullet \sim(F \vee T)] \\
 & \sim[(F) \bullet \sim(T)] \\
 & \sim[F \bullet F] \\
 & \sim[F] \\
 & T
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & (F \rightarrow F) \rightarrow \sim[\sim(\sim T \equiv \sim F) \bullet \sim F] \\
 & (T) \rightarrow \sim[\sim(F \equiv T) \bullet T] \\
 & T \rightarrow \sim[\sim(F) \bullet T] \\
 & T \rightarrow \sim[T \bullet T] \\
 & T \rightarrow \sim[T] \\
 & T \rightarrow F \\
 & F
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \sim[\sim(\sim T \bullet T) \rightarrow \sim F] \equiv \sim(T \vee \sim F) \\
 & \sim[\sim(F \bullet T) \rightarrow T] \equiv \sim(T \vee T) \\
 & \sim[\sim(F) \rightarrow T] \equiv \sim(T) \\
 & \sim[T \rightarrow T] \equiv F \\
 & \sim[T] \equiv F \\
 & F \equiv F \\
 & T
 \end{aligned}$$