Workbook Unit 5:

Calculating Truth-Values of Complex Propositions (Part II)

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Overview

In this unit, we will continue to acquire the technique of calculating truth-values of complex propositions, this time we will consider complex propositions that involve negations.

This unit

- teaches you to calculate the truth-values of arbitrarily complex propositions created by means of two-place and one-place connectives.

PowerPoint Presentation

There is a PowerPoint Presentation that accompanies this Unit. It is available on-line as a .pps and a zipped .pps file.

1. Prerequisites

There are three prerequisites for this unit. First, you need to be able to determine the main connective for complex statements created by means of two-place and one-place connectives (§1.1 is a brief reminder, you should turn back to Unit 2 for more). Second, you need to learn the basic truth tables by heart. Third, you need to be able to calculate the truth-value of an arbitrarily complex proposition created by means of two-place connectives (Unit 4).

1.1. Main Connective (Reminder)

Any given (well-formed) statement has only one main connective. When you think about how statements are constructed, the main connective is the last one that is put in.

Consider two statements

~(V • L)   ~V • L

What is the difference? Well, recall that the tilde binds what stands immediately to its right. In the first case, the tilde stands immediately next to a parenthesis, which envelops a conjunction. The tilde negates the parenthesized conjunction! The whole statement is a negation of a conjunction of V and L. The main connective is the tilde. In the second case, the tilde stands immediately next to a simple statement V so it negates V. The whole statement is a conjunction of the negation of V and of L. The main connective is the dot.

Let's think about how the statements were constructed again:

And let's put in some interpretations. Let 'V' stand for 'Joe will go to heaven' while 'L' stands for 'Joe will go to hell'. Substituting:

~(V • L)

It is not the case that Joe will go both to heaven and to hell.

~V • L

Joe will not go to heaven but he will go to hell.

As you see, the statements are very different indeed.

Example 1 and 2

When it comes to tildes, you must remember that the tilde negates what stands to its immediate right. Consider two statements:

~A • B

The first is a conjunction of a negated simple statement (A) and a simple statement B, i.e.:

~(A • B)

The second is a negation of a conjunction of two simple statements A and B:

~A • B

main connective

~A • B

~(A • B)

main connective

~(A • B)

Example 3, 4 and 5

The rule that the tilde negates what stands immediately to its right is no less true when it comes to multiple negations. Consider the next three statements:

(1)

~~A • B

Statement (1) is a conjunction of the simple statement B and a double negation of the simple statement A.

~~A • B

~~A • B

~~A • B

(2)

~~(A • B)

Statement (2) is a double negation (negation of a negation) of the conjunction of two simple statements A and B.

~~(A • B)

~~(A • B)

~~(A • B)

(3)

~(~A • B)

Statement (3) is a negation of a conjunction of a simple statement B and the negation of a simple statement A.

~(~A • B)

~(~A • B)

main connective

~(~A • B)

Example 6.

$$\sim(\sim A \vee (B \bullet C))$$

[You might want to work on your own and check your work with mine later.]

Let’s work inside out, remembering that tilde binds what is immediately to its right.

$$\sim(\underbrace{\sim A} \vee (B \bullet C))$$

The next step is to work with statements thus marked:

$$\sim(\underbrace{\underbrace{\sim A} \vee \underbrace{(B \bullet C)}})$$

Now it is clear that the main connective is the first tilde:

$$\sim(\underbrace{\underbrace{\sim A} \vee \underbrace{(B \bullet C)}})$$

main connective

$$\sim(\sim A \vee (B \bullet C))$$

This is a conjunction whose first conjunct is a negation of the disjunction ($\sim A \vee B$) and the second conjunct is itself a conjunction of a double negated A and of a disjunction of B and C.

Example 7.

$$\sim(\sim A \vee B) \bullet (\sim\sim A \bullet (B \vee C))$$

Let's work inside out, remembering that tilde binds what is immediately to its right.

$$\sim(\sim A \vee B) \bullet (\sim\sim A \bullet (B \vee C))$$

The next step is to work with statements thus marked:

$$\sim(\sim A \vee B) \bullet (\sim\sim A \bullet (B \vee C))$$

and

$$\sim(\sim A \vee B) \bullet (\sim\sim A \bullet (B \vee C))$$

Now it is clear that the main connective is the first dot:

$$\sim(\sim A \vee B) \bullet (\sim\sim A \bullet (B \vee C))$$

main connective

$$\sim(\sim A \vee B) \bullet (\sim\sim A \bullet (B \vee C))$$

This is a conjunction whose first conjunct is a negation of the disjunction of ~A and B and the second conjunct is itself a conjunction of a double negated A and of a disjunction of B and C.

Example 8.

$$\sim[\sim(\sim A \vee (B \bullet C)) \bullet \sim\sim\sim(A \bullet (B \vee C))]$$

[You might want to work on your own and check your work with mine later.]

Let's work inside out, remembering that tilde binds what is immediately to its right.

$$\sim[\sim(\sim A \vee (B \bullet C)) \bullet \sim\sim\sim(A \bullet (B \vee C))]$$

The next step is to work with statements thus marked:

$$\sim[\sim(\sim A \vee (B \bullet C)) \bullet \sim\sim\sim(A \bullet (B \vee C))]$$

Once you've marked the parentheses, you will see that they are negated, one of them three times!

$$\sim[\sim(\sim A \vee (B \bullet C)) \bullet \sim\sim\sim(A \bullet (B \vee C))]$$

Now we can bind the final parenthesis binding

$$\sim[\sim(\sim A \vee (B \bullet C)) \bullet \sim\sim\sim(A \bullet (B \vee C))]$$

main connective

$$\sim[\sim(\sim A \vee (B \bullet C)) \bullet \sim\sim\sim(A \bullet (B \vee C))]$$

You can thus see that the main connective is the first tilde – this is a negation of a huge statement, which is a conjunction whose first conjunct is a single negation (of a further compound statement $\sim A \vee (B \bullet C)$) and whose second conjunct is a triple negation (of a further compound statement $A \bullet (B \vee C)$).

1.2. Basic Truth Tables (Reminder)

You also need to know the basic truth tables by heart. Please, fill out the following truth tables and check that you have done so correctly. If not, make sure you go back to Unit 2 for further explanations.

p	q	$p \bullet q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \equiv q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

p	$\sim p$
T	
F	

2. Calculating the Truth-Values of Complex Propositions

2.1. Negations of Simple and Complex Propositions

With negated statements, you need to pay close attention as to whether the negation pertains to a simple statement or to a parenthesis. This makes a lot of difference as the following simple example illustrates.

Consider two statements:

(1) ~A • B
(2) ~(A • B)

What is the difference between (1) and (2)? Well the first is a conjunction whose first conjunct is negated. The second is a negation of a conjunction. (Remember the difference between “You will not get an A but you will get a B” and “You will not get both an A and a B”.) And there is a difference as far as the calculation of truth-values is concerned.

Suppose that A is true and B is false.

Let us calculate the truth-value of (1) first. This is a conjunction. We do not know its truth-value immediately. We do know the truth-value of the second conjunct B but not that of the first conjunct. However, we can rather quickly calculate the truth-value of ~A since we know the truth-value of A. Let’s substitute the appropriate truth-values for A and B:

~T • F

We can begin the calculation. Since the tilde attaches to a truth-value, we can already calculate it “in”, so we have:

F • F because ~T is F
F because a conjunction whose both conjuncts are false is false

Take (2) now. Let’s substitute the truth-values:

~(T • F)

This is a negation. Do we know its truth-value? Not immediately because this is a negation of the parenthesized conjunction. The tilde negates the parentheses, so we need to know the value of the statement in the parentheses first. And we can calculate:

~(T • F)
~(F) because the conjunction whose second conjunct is false, is false
T because ~F is T

In summary:

$$\underbrace{\underbrace{\sim T}_{F} \bullet F}_{F} \qquad\qquad \underbrace{\sim\underbrace{(T \bullet F)}_{(F)}}_{T}$$

Exercise Truth-Values (B) – 1

Complete the calculations. Check that your results match the results in the *Solutions* before proceeding.

1. ~T ∨ (~T → F)

2. ~(T ≡ T) ∨ ~(F ≡ F)

3. ~(T → T) ∨ (~F → F)

4. (~T • ~T) ∨ ~(F ≡ F)

5. ~T ≡ (~T ∨ (~F → F))

6. (T ≡ T) • (~T ≡ (~T → F))

7. ~(F ≡ F) → ~(F ∨ (T → F))

8. ~[(F ≡ F) ∨ ~(F ∨ F)] ≡ (T • F)

Exercise Truth-Values (B) – 2

Carry out the following calculations step-by-step, rewriting all the necessary connectives and parentheses. Do not make shortcuts. Check that your results match the results in the *Solutions* before proceeding.

1. $\sim T \vee T$

2. $\sim(T \vee T)$

3. $\sim T \vee \sim T$

4. $\sim(T \to T)$

5. $\sim T \to T$

6. $\sim T \to \sim T$

7. $\sim T \vee (\sim T \to F)$

8. $\sim T \to \sim(T \equiv F)$

9. $(\sim T \to \sim T) \vee F$

10. $\sim(T \bullet T) \to F$

11. $(\sim F \vee \sim F) \bullet (\sim T \vee \sim T)$

12. $\sim(F \vee F) \bullet \sim(F \equiv T)$

13. $\sim F \rightarrow [\sim F \bullet (\sim T \vee \sim T)]$

14. $\sim F \bullet \sim[F \vee (T \equiv T)]$

15. $[\sim T \equiv (\sim F \bullet \sim T)] \rightarrow \sim T$

16. $\sim T \bullet \sim[F \vee \sim(T \vee F)]$

Exercise Truth-Values (B) – 3

Complete the calculations. Check that your results match the results in the *Solutions* before proceeding.

1. T ∨ ~(F → ~F)

   T ∨ ~(F →)

   T ∨ ~

   T ∨

2. ~F • ~(~F ∨ ~F)

   • ~(∨)

   • ~

   •

3. ~[F ≡ ~(F ∨ T)]

   ~[F ≡ ~]

   ~[F ≡]

   ~

4. ~[T ∨ ~(F → ~F)]

   ~[T ∨ ~(F →)]

   ~[T ∨ ~]

   ~[T ∨]

   ~

5. ~T • ~(T ∨ (~F → F))

   • ~(T ∨ (→ F))

   • ~(T ∨)

   • ~

   •

6. ~(T ∨ ~F) ≡ (~T ∨ ~(T → F))

   ~(T ∨) ≡ (∨ ~)

   ~ ≡ (∨)

   ≡

Exercise Truth-Values (B) – 4

Carry out the following calculations step-by-step, rewriting all the necessary connectives and parentheses. Do not make shortcuts. Check that your results match the results in the *Solutions* before proceeding.

1. $(\sim F \bullet \sim T) \equiv \sim(T \equiv \sim F)$

2. $\sim(F \vee \sim F) \rightarrow \sim(F \vee \sim T)$

3. $\sim(\sim F \bullet \sim F) \rightarrow \sim(F \rightarrow \sim T)$

4. $\sim[(\sim F \equiv \sim F) \bullet \sim F]$

5. $\sim[(\sim F \bullet \sim F) \rightarrow \sim(F \rightarrow \sim T)]$

6. $\sim[(\sim T \equiv \sim F) \bullet \sim F] \rightarrow \sim T$

7. $\sim[\sim(T \equiv F) \bullet \sim F] \vee \sim T$

8. $\sim[\sim(\sim T \vee \sim F) \bullet \sim F] \rightarrow \sim T$

9. $\sim T \vee \sim[\sim F \bullet \sim(T \rightarrow \sim F)]$

10. $\sim\{T \vee \sim[\sim F \bullet \sim(T \rightarrow \sim F)]\}$

11. $\sim\{\sim T \equiv \sim[\sim T \vee \sim(\sim F \bullet \sim F)]\}$

12. $\sim(\sim T \vee \sim F) \equiv \sim[F \equiv \sim(F \bullet \sim F)]$

2.2. Multiple Negations

Suppose that A is true. The negation of A will be, of course, false:

```
~T
F
```

What about the negation of the negation of A? Since the negation of A is false, the negation of it (the negation of A) is true:

```
~~T
~F
T
```

Right? (Note here lies the wisdom of your grammar teachers always telling you not to use double negation in English. This is because a double negation says what you can in much simpler terms say using the doubly negated statement itself. -- But in logic we need to be able to calculate a thousand-tuple negation of a statement because we are looking for a fully general theory.

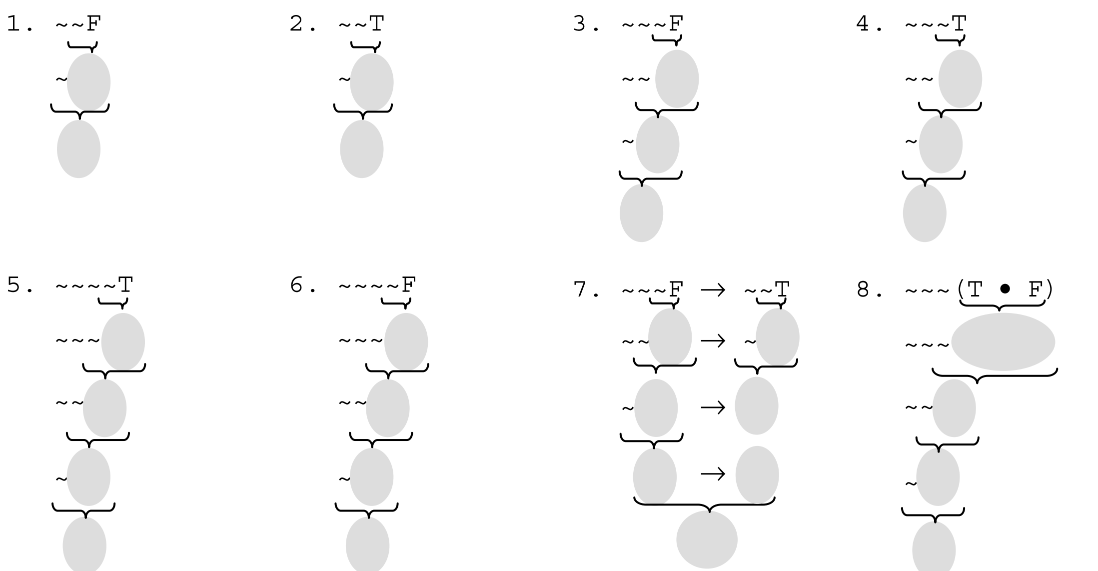
What about a triple negation?

```
~~~T
~~F
~T
F
```

And so on.

Exercise Truth-Values Multiple-Negations

Complete the calculations. Check that your results match the results in the *Solutions* before proceeding.



2.3. Multiple Negations in Complex Statements

The same rules as above apply. A tilde negates what stands immediately to the right. So, in ~A, what is negated is A, in ~(A ∨ B) – a disjunction is negated, in ~~A – the first tilde negates the negation of A (~A), in ~(~A ∨ B) – the first tilde negates the disjunction ~A ∨ B.

Contrast:

(1)	(2)	(3)
~~A • B	~~(A • B)	~(~A • B)
Statement (1) is a conjunction of a simple statement B and a double negation of the simple statement A.	Statement (2) is a double negation (negation of a negation) of the conjunction of two simple statements A and B.	Statement (3) is a negation of a conjunction of a simple statement B and the negation of a simple statement A.

The structure of these statements differs and it will be reflected in the way that the truth-values of the statements will be calculated. Suppose that A is true while B is false:

```
~~T • F
~F  • F
T   • F
   F
```

```
~~(T • F)
~~(F)
~T
F
```

```
~(~T • F)
~(F  • F)
~(F)
T
```

Exercise Truth-Values (B) – 5

Complete the following calculations. Check that your results match the results in the *Solutions* before proceeding.

```
1. ~T → ~~F
        → ~
        →
```

```
2. ~~(T ≡ F)
   ~~
   ~
```

```
3. ~(~T ≡ F)
   ~(   ≡ F)
   ~
```

```
4. ~(~T ∨ ~F)
   ~(   ∨   )
   ~
```

```
5. ~~T ∨ ~~F
   ~   ∨ ~
       ∨
```

```
6. ~(~F → F)
   ~(   → F)
   ~
```

Exercise Truth-Values (B) – 6

Complete the calculations. Check that your results match the results in the *Solutions* before proceeding.

```
1. ~~T • ~(~T • F)
   ~   • ~(   • F)
       • ~
       •
```

```
2. ~~T • (~~T • F)
   ~   • (~   • F)
       • (    • F)
       •
```

```
3. ~[~T ≡ ~(~F • ~~T)]
   ~[   ≡ ~(   • ~   )]
   ~[   ≡ ~(   •     )]
   ~[   ≡ ~           ]
   ~[   ≡             ]
   ~
```

```
4. ~~[T ≡ ~(~F • ~~T)]
   ~~[T ≡ ~(   • ~   )]
   ~~[T ≡ ~(   •     )]
   ~~[T ≡ ~           ]
   ~~[T ≡             ]
   ~~
   ~
```

```
5. ~[T → ~(T ≡ ~~~T)]
   ~[T → ~(T ≡ ~~   )]
   ~[T → ~(T ≡ ~    )]
   ~[T → ~(T ≡      )]
   ~[T → ~           ]
   ~[T →             ]
   ~
```

```
6. ~[~(~F → ~~~T) • ~~~F]
   ~[~(   → ~~  ) • ~~  ]
   ~[~(   → ~   ) • ~   ]
   ~[~(   →     ) •     ]
   ~[~            •     ]
   ~[             •     ]
   ~
```

Exercise Truth-Values (B) – 7

Carry out the following calculations step-by-step, rewriting all the necessary connectives and parentheses. Do not make shortcuts. Check that your results match the results in the *Solutions* before proceeding.

1. $\sim\sim T \vee \sim\sim F$

2. $\sim(\sim T \vee \sim F)$

3. $\sim\sim\sim F \rightarrow \sim\sim\sim\sim T$

4. $\sim\sim\sim(F \rightarrow \sim T)$

5. $\sim\sim(T \vee \sim T)$

6. $\sim\sim(\sim\sim T \rightarrow \sim T)$

7. $\sim(T \bullet \sim T) \rightarrow \sim(\sim F \vee \sim F)$

8. $\sim[(\sim T \vee \sim T) \bullet \sim(F \vee \sim F)]$

9. $(F \rightarrow F) \rightarrow \sim[\sim(\sim T \equiv \sim F) \bullet \sim F]$

10. $\sim[\sim(\sim T \bullet T) \rightarrow \sim F] \equiv \sim(T \vee \sim F)$

Exercise "Big Sentence"

At the beginning of the previous unit, I've tried to convince you that a full-proof technique of doing truth-value calculations might be useful because:

> You never know, after all, when you might encounter a teacher who would say: *You will get an A in this course if and only if the following proposition is true: "If it is both the case that London is the capital of the UK and that either Washington is the capital of the US but it is not the largest city in the US or Washington is no longer the capital of the US though it is the largest city in the US, then it is not true that London is the capital of Poland or the UK".*

Your task now is to symbolize this italicized sentence and determine whether the sentence is true or false. (I will not provide a solution to this exercise, but you might want to check with each other on the Bulletin Board, how you have done.)

A := ________________________[your name] will get an A in logic
L := London is the capital of the UK
P := London is the capital of Poland
U := Washington is the largest city in the US
W := Washington is the capital of the US

3. Summary

In this unit, you have acquired the skill of calculating the truth-value of complicated propositions (created by means of two-place and one-place connectives) given the truth-values of simple propositions.

What You Need to Know and Do

- You need be able to calculate the truth-value of any proposition using the step-by-step method introduced in this unit..