

Solutions to Workbook Exercises

Unit 4:

Calculating Truth-Values of Complex Propositions (Part I)

Exercise Truth-Values (A) – 1

$$1. \underbrace{(\underbrace{T \bullet T}_T) \vee (\underbrace{F \rightarrow F}_T)}_T$$

$$2. \underbrace{T \bullet (\underbrace{T \vee (\underbrace{F \rightarrow F}_T)})}_T$$

$$3. \underbrace{((\underbrace{T \bullet T}_T) \vee F) \rightarrow F}_F$$

$$4. \underbrace{T \bullet ((\underbrace{T \vee F}_T) \rightarrow F)}_F$$

$$5. \underbrace{(\underbrace{T \rightarrow F}_F) \bullet ((\underbrace{T \bullet F}_F) \rightarrow F)}_F$$

$$6. \underbrace{(T \equiv (\underbrace{T \vee F}_T)) \rightarrow F}_F$$

$$7. \underbrace{(\underbrace{F \rightarrow T}_T) \rightarrow (\underbrace{T \rightarrow F}_F)}_F$$

$$8. \underbrace{F \rightarrow (F \rightarrow (\underbrace{F \rightarrow F}_T))}_T$$

$$9. \underbrace{((\underbrace{F \rightarrow F}_T) \rightarrow F) \rightarrow F}_T$$

$$10. \underbrace{((\underbrace{F \rightarrow T}_T) \rightarrow F) \equiv (\underbrace{F \equiv F}_T)}_F$$

Exercise Truth-Values (A) – 2

Carry out the following calculations step-by-step, rewriting all the necessary connectives and parentheses. Do not make shortcuts. Check that your results match the results in the Solutions before proceeding.

$$\begin{aligned} 1. & (F \bullet T) \vee (F \rightarrow F) \\ & (F) \vee (T) \\ & T \end{aligned}$$

$$\begin{aligned} 6. & (F \bullet (T \vee F)) \rightarrow F \\ & (F \bullet (T)) \rightarrow F \\ & (F) \rightarrow F \\ & T \end{aligned}$$

$$\begin{aligned} 2. & F \bullet (T \vee (F \rightarrow F)) \\ & F \bullet (T \vee (T)) \\ & F \bullet (T) \\ & F \end{aligned}$$

$$\begin{aligned} 7. & (F \rightarrow F) \rightarrow (F \rightarrow T) \\ & (T) \rightarrow (T) \\ & T \end{aligned}$$

$$\begin{aligned} 3. & ((F \bullet T) \vee F) \rightarrow F \\ & ((F) \vee F) \rightarrow F \\ & (F) \rightarrow F \\ & T \end{aligned}$$

$$\begin{aligned} 8. & F \rightarrow (F \rightarrow (F \rightarrow T)) \\ & F \rightarrow (F \rightarrow (T)) \\ & F \rightarrow (T) \\ & T \end{aligned}$$

$$\begin{aligned} 4. & F \bullet ((T \vee F) \rightarrow F) \\ & F \bullet ((T) \rightarrow F) \\ & F \bullet (F) \\ & F \end{aligned}$$

$$\begin{aligned} 9. & ((F \rightarrow T) \rightarrow F) \rightarrow T \\ & ((T) \rightarrow F) \rightarrow T \\ & (F) \rightarrow T \\ & T \end{aligned}$$

$$\begin{aligned} 5. & (T \rightarrow F) \rightarrow ((T \bullet F) \rightarrow F) \\ & (F) \rightarrow ((F) \rightarrow F) \\ & (F) \rightarrow (T) \\ & T \end{aligned}$$

$$\begin{aligned} 10. & ((F \vee T) \rightarrow F) \rightarrow (F \rightarrow F) \\ & ((T) \rightarrow F) \rightarrow (T) \\ & (F) \rightarrow T \\ & T \end{aligned}$$

Exercise Truth-Values (A) – 3

In this exercise, you will be provided with the schemata for carrying out the calculations. Make sure that you understand why you proceed in this order. In the next exercises, you will be “on your own.” Check that your results match the results in the Solutions before proceeding.

$$\begin{aligned}
 1. \quad & \{ [(T \bullet T) \rightarrow F] \rightarrow (T \vee F) \} \equiv [T \bullet (T \vee F)] \\
 & \{ [T \rightarrow F] \rightarrow T \} \equiv [T \bullet T] \\
 & \{ F \rightarrow T \} \equiv T \\
 & T \equiv T \\
 & T
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & ((T \equiv T) \rightarrow (T \equiv F)) \bullet (F \rightarrow F) \vee (T \rightarrow ((T \bullet T) \equiv F)) \\
 & (T \rightarrow F) \bullet T \vee (T \rightarrow (T \equiv F)) \\
 & (F \bullet T) \vee (T \rightarrow F) \\
 & F \vee F \\
 & F
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & ((T \vee T) \bullet (F \vee F)) \rightarrow F \equiv ((T \rightarrow ((F \rightarrow F) \rightarrow T)) \rightarrow F) \\
 & (T \bullet F) \rightarrow F \equiv ((T \rightarrow (T \rightarrow T)) \rightarrow F) \\
 & (F \rightarrow F) \equiv ((T \rightarrow T) \rightarrow F) \\
 & T \equiv (T \rightarrow F) \\
 & T \equiv F \\
 & F
 \end{aligned}$$

Exercise Truth-Values (A) – 5

On the assumption that A, B and C are true while K, L and M are false, what is the truth-value of the following propositions:

$$\begin{array}{ll}
 1. (A \rightarrow M) \equiv [L \vee (B \bullet K)] & 2. A \rightarrow [L \equiv (C \vee (K \bullet B))] \\
 (T \rightarrow F) \equiv [F \vee (T \bullet F)] & T \rightarrow [F \equiv (T \vee (F \bullet T))] \\
 (F) \equiv [F \vee (F)] & T \rightarrow [F \equiv (T \vee (F))] \\
 (F) \equiv [F] & T \rightarrow [F \equiv (T)] \\
 T & T \rightarrow [F] \\
 & F
 \end{array}$$

$$\begin{array}{ll}
 3. ((K \rightarrow B) \equiv L) \vee M \bullet B & 4. ((K \rightarrow M) \equiv C) \vee (A \bullet M) \\
 ((F \rightarrow T) \equiv F) \vee F \bullet T & ((F \rightarrow F) \equiv T) \vee (T \bullet F) \\
 ((T) \equiv F) \vee F \bullet T & ((T) \equiv T) \vee (F) \\
 ((F) \vee F) \bullet T & (T) \vee (F) \\
 (F) \bullet T & T \\
 F &
 \end{array}$$

$$\begin{array}{l}
 5. [((L \vee A) \vee M) \vee B] \equiv [(L \bullet A) \bullet (M \bullet B)] \\
 [((F \vee T) \vee F) \vee T] \equiv [(F \bullet T) \bullet (F \bullet T)] \\
 [((T) \vee F) \vee T] \equiv [(F) \bullet (F)] \\
 [(T) \vee T] \equiv [F] \\
 [T] \equiv [F] \\
 F
 \end{array}$$

$$\begin{array}{l}
 6. [((K \rightarrow B) \equiv L) \rightarrow M] \equiv [((M \rightarrow C) \rightarrow M) \rightarrow M] \\
 [((F \rightarrow T) \equiv F) \rightarrow F] \equiv [((F \rightarrow T) \rightarrow F) \rightarrow F] \\
 [((T) \equiv F) \rightarrow F] \equiv [((T) \rightarrow F) \rightarrow F] \\
 [(F) \rightarrow F] \equiv [(F) \rightarrow F] \\
 [T] \equiv [T] \\
 T
 \end{array}$$