Workbook Unit 3:

Symbolizations

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1. Overview

In the previous unit, you have learned the basics of the language propositional logic. In this unit, you will continue to learn that language and in particular you will be translating English sentences into it. This task, which you already began learning, is called symbolization. It is a very difficult task indeed. Moreover, its difficulty has to do with the fact that it really is more like an art (this may be exaggerating a bit, but it is certainly a skill) and this means that there is nothing like an algorithm or a sure-fire recipe that you can learn and thereafter know how to symbolize. You will learn certain tricks or “keys” (like in guitar playing), you will learn what to look for in a sentence so that you can symbolize efficiently. But in the end, it is all about practice. As before, there will be lots of exercises.

Before this is such a practical unit, I want you to take out a pencil right now and in each of the examples, as we go along, you should right in what you think the symbolization would be, only then read on and see where you were mistaken. (You should hope that you will have been mistaken somewhere – the more mistakes you make here, the higher the chances that you won’t make them on the test.)

This unit

- teaches you to symbolize more complicated statements
- teaches you to symbolize statements that contain the connective-words ‘neither … nor …’, ‘not both … and …’, ‘either … or … but not both’, ‘unless’, and the very difficult ‘only if’
- introduces the distinction between necessary and sufficient conditions (in light of the conditionals)

PowerPoint Presentation

There is a PowerPoint Presentation that accompanies this Unit. It is available on-line as a .pps and a zipped .pps file.

2. Symbolization as an Art and as a Skill

Symbolization is a very difficult but a very useful skill. It allows you to bridge the gap between the logical theory and its applications to real-world arguments. If the later techniques you learn are to have any relevance to real life you need to learn how to symbolize.

We will proceed by means of more and more complex examples, which will be alerting you to the factors you need to watch for. As I mentioned, unfortunately there is no algorithm for symbolizations. But there are some rules of thumb that you should bear in mind:

Paraphrase	Rephrase the statement in such a way as to make the logical structure of the statement more perspicuous.
Mark connectives	Mark all of the connectives, e.g. by underlining them
Key	Construct the symbolization key, choosing letters that are easy to remember
Main Connective	Decide what the main connective is, put parentheses into the statement
Partial Symbolization	In very complicated statements, proceed step-by-step, replacing simple statements with letters.

OK. If you have your pencil ready – ready, steady, go…

Example 1

If Susie wears a new dress then either Jack or Tim will inviter her out.

The statement has a relatively clear logical structure, which will be evident when you underline all of the connectives:

If Susie wears a new dress then either Jack or Tim will invite her out.

In this way, you can see what simple statements need to be included in the symbolization key:

S: **S**usie wears a new dress
J: **J**ack will invite Susie out
T: **T**im will invite Susie out

Try to symbolize the statement using the symbolization key just constructed.

The crucial thing that we need to do, is to decide what the main connective is and place the parentheses accordingly. In our case, it seems relatively clear that this is a conditional. Think about what it says. It says roughly “If blah-blah-blah then bleh-bleh-bleh”. So, we should put in the parentheses thus:

> If Susie wears a new dress then (either Jack will invite her out or Tim will invite her out).

A good way for checking that you have not made a humongous error in placing the parentheses is that

what is inside the parentheses must always be a proposition

If what is inside the parentheses is not a sentence, you know that you have placed the parentheses wrong.

Suppose that you thought that ‘or’ is the main connective. This would yield:

> (If Susie wears a new dress then either Jack will invite her out) or Tim will invite her out.

In the parenthesis thus put the ‘either’ is “unfinished”, as it were.

I should say, however, that the requirement that the parentheses contain a statement is a necessary but not a sufficient condition for placing the parentheses right. This means that if at least one of your parentheses does not contain a statement, you can be sure that you have placed them wrong. But you cannot be sure that you placed them right, if all of your parentheses do contain statements.

We can now substitute letters from the symbolization key into the statement:

> If S then (either J or T)

And now all that remains is substituting the symbols for the connectives:

$$S \rightarrow (J \vee T)$$

One good rule to learn early is to read back the symbolization using the symbolization key and checking whether indeed your symbolization says the same thing as your original statement. (This is particularly important for more complicated symbolizations.)

Example 2

Let us consider a variation on the statement we have just symbolized. Underline all the connectives; construct a symbolization key; try to symbolize:

> If Susie wears a new dress and will no longer fret then either Jack or Tim will invite her out.

S: **S**usie wears a new dress
F: Susie will **f**ret
J: **J**ack will invite Susie out
T: **T**im will invite Susie out

Again, the crucial thing is to decide what the main connective is and place the parentheses accordingly. It should seem relatively clear to you that the statement is once again a conditional. It says "If Susie blah-blah-blah then bleh-bleh-bleh". So, the parentheses should be put thus:

> If (Susie wears a new dress and Susie will no longer fret) then (either Jack will invite her out or Tim will inviter her out).

Check that indeed this is the only way to put the parenthesis in:

> Exercise:
> Put the parentheses in such a way as if 'and' was the main connective.
>
> > If Susie wears a new dress and Susie will no longer fret then either Jack will invite her out or Tim will inviter her out.
>
> Put the parentheses in such a way as if 'or' was the main connective.
>
> > If Susie wears a new dress and Susie will no longer fret then either Jack will invite her out or Tim will inviter her out.
>
> In either case, you will see that at least one of the parentheses will not contain a statement.

Substitute simple statement with letters from the symbolization key:

> If (S and not F) then (either J or T)

and connective-phrases with respective symbols:

> $(S \bullet \sim F) \rightarrow (J \vee T)$

Example 3

> Either Susie will go out with both Jack and Tim or they will both invite Ann.

The fact that this statement is a disjunction is perhaps more clear than the fact both its disjuncts (in particular the second) are conjunctions. Let us go step by step and begin by underlining all of the connective-phrases:

> Either Susie will go out with both Jack and Tim or they will both invite Ann.

If you think that you can construct the symbolization key and the symbolization, do so now (Hint: there are four simple statements):

Since it is relatively easy to see that the statement is a disjunction, let us put the parentheses in:

> Either (Susie will go out with both Jack and Tim) or (they will both invite Ann).

(As before, you can try to treat the other connectives as the main connectives, but you will see that when you put the parentheses another way, you will not get statements inside the parentheses.)

Now let us look inside the parentheses. Both of them contain conjunctions. Let us expand them so we are quite clear what simple statements are being conjoined.

> Either (both Susie will go out with Jack and Susie will go out with Tim) or (both Jack will invite Ann and Tim will invite Ann).

Now at last we can clearly see the simple statements that our statement is constructed from. We can construct the symbolization key:

S: **S**usie will go out with Jack
U: S**u**sie will go out with Tim
J: **J**ack will invite Ann
T: **T**im will invite Ann

(Note that you may have used different letters above. The choice of letters is arbitrary as long as you obey the rules laid out in the previous unit.)

We are thus ready to do the partial symbolization:

> Either (both S and U) or (both J and T)

and complete the symbolization:

$$(S \bullet U) \vee (J \bullet T)$$

Example 4

> If Susie goes out with Jack or Tim then either Jack or Tim will not invite Ann out.

Do check that we can here use the same symbolization key as above. Try to do the symbolization yourself.

S: **S**usie will go out with Jack
U: S**u**sie will go out with Tim
J: **J**ack will invite Ann
T: **T**im will invite Ann

Let's go step-by-step. First, underline all occurrences of the connective-phrases:

> If Susie goes out with Jack or Tim then either Jack or Tim will not invite Ann out.

It should seem relatively clear to you that the statement is a conditional. (If it is not clear, try deciding what other connective you think is the main one and then if you place the parentheses around the component statements you will see that they are not statements.)

> If (Susie goes out with Jack or Tim) then (either Jack or Tim will not invite Ann out).

The disjunction in the antecedent of the conditional is relatively straightforward, we can symbolize it partially thus:

> $(S \vee U) \rightarrow$ (either Jack or Tim will not invite Ann out)

But let us pause not to make a mistake in the symbolization of the disjunction in the consequent of the conditional. There are two connectives here: ‘or’ and ‘not’. What you have to decide is what exactly is being said. You have to ask yourself what the English phrase “either Jack or Tim will not invite Ann out” means. There are two options:

(a) either Jack will not invite Ann out or Tim will not invite Ann out
(b) either Jack will invite Ann out or Tim will not invite Ann out

If you have a clear mind, you will have no problem in deciding that the phrased used in the original statement actually means the same as (a). This is why we allow ourselves to shorten the sentence in this way. If what we meant to say were (b), we would actually have to say something close to the way in which (b) is phrased. So, this means that we actually are dealing with two negations, not just one:

> $(S \vee U) \rightarrow$ (either Jack will not invite Ann out or Tim will not invite Ann out)

We can complete the symbolization thus:

> $(S \vee U) \rightarrow (\sim J \vee \sim T)$

The Disambiguating Force of ‘Either … or…’, ‘Both … and …’, ‘If … then…’

Let us pause to reflect a little. There are a number of connectives in English that could be phrased just by using one word, like ‘or’, ‘and’, ‘if’:

> Susie will go out with Tim **or** Ann will go out with Jack
> Susie will go out with Tim **and** Ann will go out with Jack
> Susie will go out with Tim **if** Ann goes out with Jack

or they can be expressed using a double phrase like ‘either… or…’, ‘both… and…’, ‘if… then…’:

> **either** Susie will go out with Tim **or** Ann will go out with Jack
> **both** Susie will go out with Tim **and** Ann will go out with Jack
> **if** Ann goes out with Jack **then** Susie will go out with Tim

When the statements are relatively uncomplicated in structure, it is often not important whether single-word or double-word phrases are used. But when the structure of the statements becomes complicated, the double-word phrases help tremendously in letting us know what is being said (i.e. what the main connective is).

The statement:

> Susie will go out with Tim **or** Ann will go out with Jack **and** Betty will go out with Dick

is ambiguous between (and note that to express what it is ambiguous between we will be using the disambiguating 'both' and 'either'):

> **Either S**usie will go out with Tim **or both A**nn will go out with Jack **and B**etty will go out with Dick
> $S \vee (A \bullet B)$
> **It is both the case that either S**usie will go out with Tim **or A**nn will go out with Jack **and that B**etty will go out with Dick
> $(S \vee A) \bullet B$

Ex. Disambiguation

Consider the following ambiguous sentences. Try to phrase them in such a way as to disambiguate them. Then symbolize them.

(a) Abe will read a couple of textbooks or listen to some lectures and solve some problems.

1:

2:

L: Abe will listen to some lectures
R: Abe will read some textbooks
S: Abe will solve some problems

[1]

[2]

(b) If Ann finishes her graduate studies then she will work as a scientist or she will become a teacher.

1:

2:

G: Ann finishes her graduate studies
S: Ann will work as a scientist
T: Ann will become a teacher

[1]

[2]

(c) Ann will finish her graduate studies and she will work as a scientist or she will become a teacher if she can live with little pay.

1:

2:

3:

4:

5:

G: Ann finishes her graduate studies
L: Ann can live with little pay
S: Ann will work as a scientist
T: Ann will become a teacher

[1]

[2]

[3]

[4]

[5]

Ex. Symbolization 1

Symbolize the following statements:

D: Ann diets	**F**: Ann is fat	**I**: Billy diets
E: Ann exercises	**H**: Ann is healthy	**O**: Billy jogs
S: Ann swims		**T**: Billy is fat
J: Ann jogs		

(a) If Ann does not exercise, she will get fat.

(b) If Ann either diets or exercises, she will get healthier.

(c) Ann will either diet and swim or she will diet and jog.

(d) Ann will diet and she will either swim or jog.

(e) If Ann swims then she will not jog.

(f) Ann will be healthy if she both diets and either swims or jogs.

(g) Ann will be healthy just in case both she and Billy will jog.

(h) Billy will jog if but only if either Ann jogs or exercises

(i) Provided that Billy and Ann are on a diet, they will both be jogging.

(j) Ann will either swim or jog provided that Billy either jogs or is on a diet.

(k) If either Ann or Billy are getting fat that if Ann does not diet then Billy will not diet.

(l) Assuming that Ann and Billy are both on a diet, Ann will jog when and only when Billy jogs.

(m) Either Ann and Billy will diet or they will both jog.

(n) If either Ann and Billy both diet or they both jog then if Ann is not getting fat then Billy won't be getting fat.

Example 5: The Main Connective Determined by Meaning

We will now be turning to some more complicated examples.

> If Susie goes out with Jack then Tim will invite Ann but if Susie goes out with Tim then Jack will invite Ann.

Again we can use the same symbolization key as above. Try to do the symbolization yourself.

S: **S**usie will go out with Jack
U: S**u**sie will go out with Tim
J: **J**ack will invite Ann
T: **T**im will invite Ann

Let's underline all occurrences of the connective-phrases:

> If Susie goes out with Jack then Tim will invite Ann but if Susie goes out with Tim then Jack will invite Ann.

Here the determination of what the main connective is will not be mechanical. This is really where the thought that symbolization is an art starts becoming manifest. There are at least two ways in which the parentheses could be placed without violating the statement-in-parentheses requirement. But in fact it is clear to anyone who hears the statement that 'but' is the main connective. We are saying something of the shape "blah-blah-blah but bleh-bleh-bleh". In fact, when you read the statement out loud, with understanding, you will *have* to put emphasis on the 'but'. Otherwise, you will not have expressed the intention behind the statement.

> (If Susie goes out with Jack then Tim will invite Ann) but (if Susie goes out with Tim then Jack will invite Ann)

Perhaps to emphasize the point that 'but' is the main connective, you can reformulate the statement in this fashion to convince yourself that this is indeed what is being said:

> It is both the case that (if Susie goes out with Jack then Tim will invite Ann) and that (if Susie goes out with Tim then Jack will invite Ann)

This time, once we decided what the main connective is, the rest is easy:

> (if S then T) and (if U then J)
> $(S \rightarrow T) \bullet (U \rightarrow J)$

Example 6: The Main Connective Determined by Meaning

Here is another example where it is the meaning of the statement made that determines what the main connective is.

> Susie is responsible for her action just in case she actually committed the act and she either intended or desired to commit it.

Try to do the symbolization yourself using the following symbolization key (those of you who already know a little about predicate logic will realize that the symbolization key that is available in propositional logic does not and cannot capture the whole sense of the statement; we will work using this key treating it as a simplification):

C: **S**usie committed the act
D: **S**usie desired to commit the act
I: **S**usie intended to commit the act
R: **S**usie is responsible for the act

Let's underline all occurrences of the connective-phrases:

> Susie is responsible for her action just in case she actually committed the act and she either intended or desired to commit it.

As before, if you really think about what is being said you will have no problem in deciding that the biconditional here is the main connective rather than the conjunction. When you read the statement you will be emphasizing 'just in case', and this is a good, though not sure-fire, guide to what the main connective is.

Once you decided on the main connective, the rest is relatively simple:

> Susie is responsible for her action just in case (she actually committed the act and she either intended or desired to commit it)

However, you now have a complex statement within the parentheses. Here, however, there is no other way of finding the main connective. The occurrence of 'either' disambiguates the statement:

> Susie is responsible for her action just in case (she actually committed the act and (she either intended or desired to commit it))

Substituting

> R just in case (C and (I or D))
> $R \equiv (C \bullet (I \vee D))$

Examples 7 & 8: The Main Connective Determined by Comma Placement

(7) If Jung's theory is false then Freud's theory is true, on the condition that Adler's theory is false.
(8) If Jung's theory is false, then Freud's theory is true on the condition that Adler's theory is false.

These two statements differ only in the way in which the comma is placed. In English the placement of the comma is very often indicative of what the main connective is. Let's place the parentheses as indicated by the comma:

(7) (If Jung's theory is false then Freud's theory is true) on the condition that Adler's theory is false
(8) If Jung's theory is false then (Freud's theory is true on the condition that Adler's theory is false)

Try to do the symbolization yourself, given the following symbolization key:

A: **A**dler's theory is true
J: **J**ung's theory is true
F: **F**reud's theory is true

Let's underline all occurrences of the connective-phrases:

(7) (If Jung's theory is false then Freud's theory is true) on the condition that Adler's theory is false
(8) If Jung's theory is false then (Freud's theory is true on the condition that Adler's theory is false)

Since the connectives do not appear in their standard forms, we will need to paraphrase the statements to have the negations and the conditionals appear in the standard forms. Let's begin with negations, where it will be easiest to do a partial symbolization:

(7) (If ~J then F) on the condition that ~A
(8) If ~J then (F on the condition that ~A)

Now let's turn to the 'on the condition that'. You should remind yourself that whenever we say '*p* on the condition that *q*', '*q*' is the condition on which something is true, so the phrase means the same as 'if *q* then *p*' (this is something you should have under your belt from last unit; if you don't you need to do more of the on-line exercises):

(7) If ~A then (if ~J then F)
(8) If ~J then (if ~A then F)

All that remains is to do symbol substitutions:

[7] $\sim A \rightarrow (\sim J \rightarrow F)$
[8] $\sim J \rightarrow (\sim A \rightarrow F)$

Ex. Symbolization 2

Symbolize the following statements:

D: Ann diets	**F**: Ann is fat	**J**: Ann jogs	**I**: Billy diets
E: Ann exercises	**H**: Ann is healthy	**S**: Ann swims	**O**: Billy jogs
			T: Billy is fat

(a) If Ann swims, then she will not jog though she will diet.

(b) If Ann swims then she will not jog, but she will diet.

(c) If Ann swims then she will not jog, and if she jogs then she will not swim.

(d) If Ann is on a diet then Billy will be on a diet, but he will not jog.

(e) If Ann is on a diet, then Billy will be on a diet but he will not jog.

(f) Ann will jog just in case Billy jogs, and Billy will go on a diet just in case Ann goes on a diet.

(g) If Ann jogs, then she will not be getting fat provided that she goes on a diet.

(h) If Ann diets then she will not be getting fat, assuming that she is healthy.

Ex. Symbolization 3

A: Ann is on a diet	**L:** Larry is getting fat
B: Betty is on a diet.	**M:** Martin is getting fat
C: Charlie is on a diet	**N:** Newt is getting fat

(a) Either Ann is on a diet or Betty and Charlie are both on a diet.

(b) It is both the case that either Ann or Betty is on a diet and that Charlie is on a diet.

(c) Either Ann or Betty is on a diet, and in any event Charlie is on a diet.

(d) Either Larry and Martin are getting fat or Martin and Newt are getting fat

(e) Either Ann or Betty is on a diet; however, it is also the case that either Betty or Charlie is a on a diet.

(f) Either both Larry and Martin are not getting fat or Newt is not getting fat.

3. A Variety of Symbolization “Tricks”

In the following sections, you will be learning a number of symbolization “tricks.” It is important that you do the exercises for them *now*. If something seems difficult to you even after you have done the exercises, turn to the on-line exercises. (If you would like to see more exercises on a given topic, let me know.) These symbolization “tricks” become crucial when you turn to more complicated symbolizations.

3.1. *n*-place Conjunctions and Disjunctions

This section might or might not be obvious so it is best to briefly make it explicit. We have introduced both conjunction and disjunction as two-place connectives. This means that ‘and’ and ‘or’ can only bind two statements. However, in ordinary language we often let ‘or’ and ‘and’ bind more than two statements, in which case we do not repeat the connective but use a comma. Consider the following statement:

(1) Ann, Betty and Charlie are on a diet.

Using the symbolization key from the above exercise we can capture the statement but we need to render it either as:

[1a] (A • B) • C
[1b] A • (B • C)

Since conjunction is a two-place connective we need to put the parentheses in. Whether we do it like in [1a] or in [1b] does not matter. If the lists are longer, there will be more choices on how to put the parentheses.

Note, however, that while it is arbitrary how the parentheses are placed around statements of the same time, once a different connective appears, the arbitrariness is gone. There is only one way to symbolize “Either Ann and Betty are on a diet or Charlie is”.

3.2. ‘Neither…nor…’, ‘Not both… and …’, ‘Both not … and …’

Now that you’ve got your feet wet in doing more complicated symbolizations, it is time for you to learn some of the symbolization tricks I mentioned at the outset. We will begin with three connective phrases that can be symbolized by means of negation and conjunction or negation and disjunction. It will be important for you to understand that the symbolizations are indeed intuitive. But thereafter you need to memorize the symbolizations by doing the exercises. (There are also on-line exercises to help you with the latter task.)

I said that all of these connective phrases can be symbolized by means of negation and conjunction, but they can also equivalently be symbolized by means of negation and disjunction. Since I believe that the former symbolizations are more intuitive, I will begin with them.

3.2.1. ‘Not both *p* and *r*’ as a negation of a conjunction

Suppose that a nice kitchen lady says to Ann:

You can have both the banana and the cake.

Given the symbolization key:

B: Ann can have the banana.
C: Ann can have the cake.

what the nice kitchen lady says can be symbolized as:

B • C

Now, soon after the nice kitchen lady said that, her nasty superior storms in and thunders grabbing Ann’s arm:

(1) You can *not* have both the banana and the cake.

What the nasty kitchen lady says is simply a denial of what the nice one said:

[1] ~(B • C)

This provides a general recipe for symbolizing all statements that have the “not both … and …” form. Consider the following examples:

(2) John will not both become a doctor and lawyer.

Given the symbolization key:

D: John is a doctor
L: John is a lawyer

statement (2) can be symbolized as:

[2] ~(D • L)

Similarly:

(3) Ann will not marry both Jim and Tim.

can be rendered as:

[3] ~(J • T)

given the symbolization key:

J: Ann will marry Jim
T: Ann will marry Tim

In general, any statement of the form "not both *p* and *r*" can be represented as "~(p • r)", though there will be also another way of representing those statements.

3.2.2. 'Neither *p* nor *r*' as a conjunction of negations

Suppose that John's mother-in-law says to John:

(1) You are neither a doctor nor a lawyer.

What is she saying?

Is she saying that John is doctor?	❍yes	❍no
Is she saying that John is a lawyer?	❍yes	❍no

I'm quite confident that you answered correctly – you are bound to if you understand what *neither nor* means. She is saying that John is *not* a doctor and she is saying that he is *not* a lawyer. In other words, what she says can be captured in terms of a conjunction of two negations thus:

John is not a doctor and John is not a lawyer.

Given the symbolization key:

D: John is a doctor
L: John is a lawyer

her statement (1) can be symbolized as:

[1] ~D • ~L

Suppose that Jennifer looks into the fridge at a black banana shape and thinks to herself:

(2) Yuck, I will **neither** eat this banana raw **nor** make a cake with it.

What is Jennifer saying? Answer the following questions given (1):

Will Jennifer will eat this banana raw?	❍yes	❍no
Will Jennifer make a cake with this banana?	❍yes	❍no

Again I'm quite sure that you answered negatively both times. Jennifer is saying both that she will not eat the banana raw and that she will not make a banana cake with it. Given the symbolization key:

C: Jennifer make a cake with this banana
R: Jennifer will eat this banana raw

we can represent statement (2) as conjunction of two negations thus:

[2] ~R • ~C

Similarly:

(3) It turned out that Ann will marry **neither** Jim **nor** Tim.

can be rendered as:

[3] ~J • ~T

given the symbolization key:

J: Ann will marry Jim
T: Ann will marry Tim

In general, any statement of the form "neither *p* nor *r*" can be represented as "~p • ~r", though there will be also another way of representing those statements.

'Both not' as a conjunction of negations

Note that sometimes the same content as that expressed by means of 'neither… nor…' can be expressed by means of 'both not'.

(1) Ann and Betty both don't have a cat.

What are we saying?

Does Ann have a cat?	❍yes	❍no
Does Betty have a cat?	❍yes	❍no

Again, I'm quite confident that you negatively both times. In other words, the content of what we are saying in (1) can be captured thus:

Ann does not have a cat and Betty does not have a cat.

Given the symbolization key:

A: Ann has a cat
B: Betty has a cat

statement (1) can be symbolized as:

[1] ~A • ~B

This looks like a symbolization pattern for 'neither…nor…' and indeed we can rephrase (1) as "Neither Ann nor Betty have a cat". These are two equivalent ways of saying the same thing.

Be careful however! While statements of the form "both not p and not r" are equivalent to statements of the form "neither p nor r"; they are not equivalent to the statements of the form "not both p and r"!

Exercise Neither-Nor, Not-both – 1

Provide the symbolizations of the following statements using the provided symbolization key:

A: Ann is on a diet	D: Dirk is on a diet
B: Betty is on a diet.	E: Evelyn is on a diet.
C: Charlie is on a diet	F: Frank is on a diet

(a) Ann and Betty are both on a diet

(b) Ann and Charlie are not both on a diet.

(c) Evelyn and Frank are both not on a diet.

(d) Neither Dirk nor Charlie are on a diet.

(e) Ann and Dirk are not both on a diet.

(f) Betty and Frank are both not on a diet.

(g) Neither Frank nor Evelyn are on a diet.

(h) Ann is on a diet but neither Betty nor Charlie is on a diet.

(i) Betty and Charlie are both not on a diet though Ann is on a diet.

(j) Ann is on a diet but not both Betty and Evelyn are on a diet

(k) If neither Betty nor Evelyn is on a diet then Charlie and Frank are not both on a diet.

(l) If Betty and Evelyn are not both on a diet then Charlie and Frank are both not on a diet.

(m) Neither Ann nor Betty is on a diet if and only if Charlie and Dirk are not both on a diet.

(n) If Ann and Betty both are not on a diet and Evelyn is not on diet then neither Charlie nor Dirk is on a diet.

(o) If Ann and Betty are not both on a diet then either Evelyn is not on diet or Charlie and Dirk are not both on a diet.

3.2.3. ‘Neither *p* nor *r*’ as a negation of a disjunction

Now that you have learned to symbolized ‘neither…nor…’ statements as a conjunction of negations, you will learn to symbolize it equivalently as a negation of a disjunction. This is captured in one of the *de Morgan laws*:

Neither *p* nor *r*	$\sim p \bullet \sim r$	$\sim(p \vee r)$

This is quite intuitive as you can see on the examples we have looked at. John’s mother-in-law said to John:

(1) You will be **neither** a doctor **nor** a lawyer.

which given the symbolization key

D: John is a doctor
L: John is a lawyer

we symbolized as

[1] $\sim D \bullet \sim L$

Note that she could have also said:

(1′) You wo**n’t** become **either** a doctor **or** a lawyer.

which would be most naturally rendered as:

[1′] $\sim(D \vee L)$

Jennifer looking into the fridge at a black banana shape thought to herself:

(2) Yuck, I will **neither** eat this banana raw **nor** make a cake with it.

but she could have had an equivalent thought:

(2′) Yuck, I wo**n’t either** eat this banana raw **or** make a cake with it.

Given the symbolization key:

C: Jennifer make a cake with this banana
R: Jennifer will eat this banana raw

we can represent her statements, respectively, as:

[2] $\sim R \bullet \sim C$
[2′] $\sim(R \vee C)$

Similarly:

(3) Ann will marry **neither** Jim **nor** Tim.

can be expressed equivalently:

(3′) Ann won’t marry **either** Jim **or** Tim.

Those statements can be symbolized, respectively, as:

[3] $\sim J \bullet \sim T$
[3′] $\sim(J \vee T)$

given the symbolization key:

J: Ann will marry Jim
T: Ann will marry Tim

3.2.4. 'Not both *p* and *r*' as a disjunction of negations

The less intuitive of the *de Morgan laws* concerns the symbolization of "not both… and …" type of statements.

Not both *p* and *r*	$\sim(p \bullet r)$	$\sim p \vee \sim r$

Consider a case where the equivalence is intuitive. Suppose that someone says:

(1) Adler's and Jung's theory cannot both be true.

Given the symbolization key:

A: Adler's theory is true
J: Jung's theory is true

we know that we can represent statement (1) thus:

[1] $\sim(A \bullet J)$

The de Morgan equivalence tells us that we can also represent the statement thus:

[1′] $\sim A \vee \sim J$
(1′) Either Adler's or Jung's theory is false.

If you think about it, this is indeed all that someone who says (1) commits herself to. To say that Adler's and Jung's theory cannot both be true is to say that at least one of them must be false: either Adler's theory or Jung's theory must be false.

Note that statement (1) can be said by someone who does not believe that either of the theories is true. Most contemporary psychologists believe that neither Adler's nor Jung's theory are true, but they can express a statement like (1), fully believing it. When they say, they are merely saying that the theories are incompatible with one another – they cannot be both true, at least one of them is false (or possibly both, as they in fact believe, are false).

The second of the de Morgan equivalence does not capture our intuitions as much as the first. I therefore suggest that you memorize them!

Neither *p* nor *r*	$\sim p \bullet \sim r$	$\sim(p \vee r)$
Not both *p* and *r*	$\sim(p \bullet r)$	$\sim p \vee \sim r$

You must memorize de Morgan equivalences!

Exercise Neither-Nor, Not-both – 2

Provide two equivalent symbolizations of the following statements using the provided symbolization key:

A: Ann is on a diet
B: Betty is on a diet.
C: Charlie is on a diet
D: Dirk is on a diet
E: Evelyn is on a diet.
F: Frank is on a diet

(a) Not both Charlie and Ann are on a diet.		
(b) Neither Ann nor Betty are on a diet.		
(c) Ann and Betty are not both on a diet.		
(d) Neither Betty nor Evelyn are on a diet.		
(e) Charlie and Frank are both not on a diet.		
(f) Neither Ann nor Betty are on a diet.		
(g) Both Charlie and Frank are not on a diet.		
(h) Betty and Evelyn are not both on a diet.		
(i) Neither Betty nor Evelyn is on a diet though Ann is on diet.		
(j) If Ann is on a diet then Betty and Charlie are not both on a diet.		
(k) Either Ann is not on a diet or Betty and Charlie are not both on a diet.		
(l) It is not the case that neither Ann nor Betty is on a diet		
(m) It is not the case that Charlie and Dirk are not both on a diet.		
(n) It would be a lie to say that Evelyn and Ann are both not on a diet.		
(o) It is neither the case that Ann is not on a diet nor that Betty is not on a diet.		

Exercise Neither-Nor, Not-both – 3

In this exercise, you will be given the same statements as you were given in Exercise Neither-Nor, Not-both – 1. This time you should only provide the symbolizations in terms of negations and disjunctions.

A: Ann is on a diet	D: Dirk is on a diet
B: Betty is on a diet.	E: Evelyn is on a diet.
C: Charlie is on a diet	F: Frank is on a diet

(b)	Ann and Charlie are not both on a diet.	
(c)	Evelyn and Frank are both not on a diet.	
(d)	Neither Dirk nor Charlie are on a diet.	
(e)	Ann and Dirk are not both on a diet.	
(f)	Betty and Frank are both not on a diet.	
(g)	Neither Frank nor Evelyn are on a diet.	
(h)	Ann is on a diet but neither Betty nor Charlie is on a diet.	
(i)	Betty and Charlie are both not on a diet though Ann is on a diet.	
(j)	Ann is on a diet but not both Betty and Evelyn are on a diet	
(k)	If neither Betty nor Evelyn is on a diet then Charlie and Frank are not both on a diet.	
(l)	If Betty and Evelyn are not both on a diet then Charlie and Frank are both not on a diet.	
(m)	Neither Ann nor Betty is on a diet if and only if Charlie and Dirk are not both on a diet.	
(n)	If Ann and Betty both are not on a diet and Evelyn is not on diet then neither Charlie nor Dirk is on a diet.	
(o)	If Ann and Betty are not both on a diet then either Evelyn is not on diet or Charlie and Dirk are not both on a diet.	

3.3. The Exclusive Disjunction 'Either… or… but not both'

When introducing the disjunction in the last unit, we said that our intuitions pull both ways – sometimes toward an inclusive disjunction (which we represent by means of our '∨'), sometimes toward an exclusive disjunction. I said then that there is a way of expressing the exclusive disjunction by means of the inclusive disjunction.

Suppose Billy's mother says to him:

(1) You can have either a cat or a dog but you can't have both of them.

This is a way of making explicit that an exclusive disjunction is intended. We can represent it now given that we have learn how to symbolize "not both" phrases. We can paraphrase statement (1) to make its structure clearer:

> Billy can have either a cat or a dog but he cannot have both a cat and a dog.

When we underline all the connectives it will become clear that 'but' is the main connective:

> (Billy can have <u>either</u> a cat <u>or</u> a dog) <u>but</u> (he can<u>not</u> have <u>both</u> a cat <u>and</u> a dog)

Given the symbolization key

C: Billy can have a cat
D: Billy can have a dog

we can symbolize (1) as:

[1] $(C \vee D) \bullet \sim(C \bullet D)$

In general,

Either *p* or *r* but not both	$(p \vee r) \bullet \sim(p \bullet r)$

Note that the exclusive disjunction is symbolized as a conjunction of the inclusive disjunction and a negation of a conjunction.

Exercise Exclusive-Disjunction

Symbolize the following statement given the symbolization key provided

A: Ann is on a diet C: Charlie is on a diet
B: Betty is on a diet. D: Dirk is on a diet

(a) Ann or Betty are on a diet but not both.

(b) Betty or Charlie are on a diet but not both.

(c) Either Charlie or Dirk is on a diet but not both.

(d) If Ann or Charlie are on a diet though not both then either Betty or Dirk are on a diet.

3.4. 'Unless'

Consider what the following statement means (aside from family trouble, obviously):

(1) I will divorce you **unless** *you change*.

The content of the statement can be expressed in two different, though as it turns out, logically equivalent ways:

(2) **If** *you do* **not** *change* **then** I will divorce you.
(3) **Either** *you change* **or** I will divorce you.

Given the symbolization key:

C: You will change
D: I will divorce you

we can symbolize the statements, respectively, as:

[2] $\sim C \rightarrow D$
[3] $C \vee D$

Two points are worth noting. First, if we render statement (1) as the implication [2], then the statement following 'unless' clause (italicized above; I'll refer to it simply as the italicized statement) becomes negated; if we render (1) as the disjunction [3], that statement is not negated. Second, in both cases [2] and [3] the italicized statement "travels" from being the second term to being a first term: in the case of the conditional [2], the italicized statement becomes the antecedent of the conditional, while in the case of the disjunction [3], the italicized statement becomes the first disjunct.

Consider another example:

(4) I will not tell you what happened **unless** *you shut up*.

Again, there are two equivalent ways of understanding the statement:

(5) **If** *you* **do not** *shut up* **then** I will not tell you what happened
(6) **Either** *you shut up* **or** I will not tell you what happened

Given the symbolization key:

S: You will shut up
T: I will tell you what happened

we can symbolize the statements, respectively, as:

[2] $\sim S \rightarrow \sim T$
[3] $S \vee \sim T$

In general:

r unless p	$p \vee r$	$\sim p \rightarrow r$

Exercise Unless

Symbolize the following statements in two equivalent ways.

A: Ann will go on a diet	D: Ann's doctor objects to Ann going on a diet
B: Betty will go on a diet.	E: Evelyn forbids Betty to go on a diet
C: Charlie will go on a diet	F: Frank will go on a diet

(a) Ann will not go on a diet unless her doctor objects to her going on a diet.

(b) Betty will not go on a diet unless Evelyn forbids her to do so.

(c) Charlie will go on a diet unless Frank goes on a diet.

(d) Ann will not go on a diet unless Betty goes on a diet.

(e) Unless Ann goes on a diet, Betty will not go on a diet.

(f) Betty and Ann will go on a diet unless Ann's doctor objects to Ann's going on a diet.

(g) Charlie and Frank will both not go on a diet unless both Betty and Ann go on a diet.

(h) Either Ann or Betty will not go on a diet unless either Charlie or Frank go on a diet.

(i) Ann will go on a diet just in case Betty goes on a diet, unless Ann's doctor objects to Ann's going on a diet.

(j) Ann will go on a diet unless Frank does not go on a diet.

3.5. Your worst nightmare: 'If' vs. 'Only if'

There are few phrases as confusing as 'only if'. You should read what is said below with understanding and then learn the symbolization schema by heart. You will thank me.

3.5.1. 'If' – A Reminder

To really appreciate how confusing 'only if' is, let us remind ourselves of the way in which we would symbolize a statement of the form "*r* if *p*". Let's do it on an example.

(1) Jane will go out with Ken **if** *he asks her politely*.

Since the italicized statement is the condition, it belongs in the antecedent of a conditional. We should thus put (1) into a standard form thus:

If *Ken asks Jane politely* then she will go out with him.

Given the symbolization key:

A: Ken asks Jane politely
G: Jane will go out with Ken

we can symbolize the statement thus:

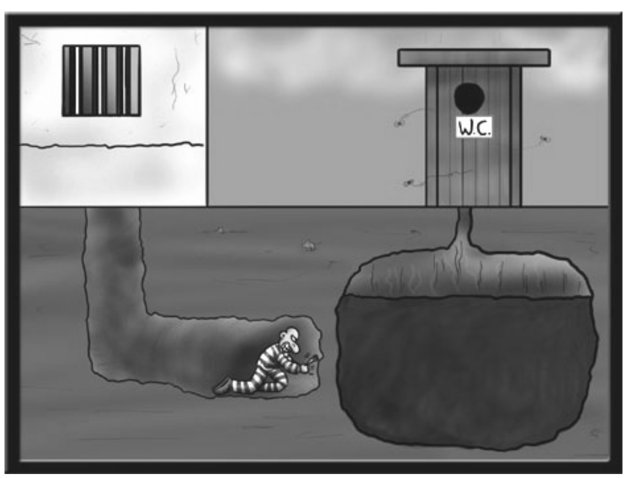
G if A
[1] $A \rightarrow G$

This will be the way to symbolize 'if' statements in general – what follows the 'if' belongs in the antecedent of the conditional.

r if *p*	$p \rightarrow r$

3.5.2. 'Only If'

All of this is fine but I have to burden you with 'only if'. No one is ever ready for 'only if'. I have found a cartoon that depicts what you are about to experience – after this already tiring unit. So take a break, and a deep breath, before you go on.



Let us consider some intuitive "only if" statements.

You will win the lottery **only if** *you buy the ticket*.
G.B. is a mother **only if** *G.B. is a woman*.
It rains **only if** *it is cloudy*.

Example 1

Let us start with the first example:

(1) You will win the lottery **only if** *you buy the ticket*.

Statement (1) is certainly true – you can win a lottery only if you buy the ticket. Without a ticket you won't win the lottery. It is natural mistake (which you have to watch out for!) to think that what follows the 'if' word is an antecedent of the conditional (just as it was above in case of the statements of the form "*r* if *p*"). Let's try put down the sentence that results from making the mistake; read it carefully and compare it to the original statement:

If *you buy the ticket* **then** you will win the lottery**.**

This statement is evidently false! Buying the ticket is certainly not sufficient for winning the lottery (we all wish it were, but it ain't). But our original statement (1) was true! So how can we say what (1) says? Well, there are two equivalent ways of saying what (1) means:

(1a) **If** *you* **do not** *buy the ticket* **then** you will **not** win the lottery.
(1b) **If** you won the lottery, **then** [this must mean that] *you bought the ticket*.

since only if you buy the ticket can you win the lottery! Given the symbolization key:

T: You will buy the lottery ticket.
W: You will win the lottery.

[1a] $\sim T \rightarrow \sim W$
[1b] $W \rightarrow T$

Example 2

(2) G.B. is a mother **only if** *G.B. is a woman*.

M: G.B. is a mother
W: G.B. is a woman.

This statement is once again obviously true (only women are mothers after all), but the sentence resulting from the natural mistake of assuming that the italicized sentence belongs in the antecedent is again evidently false:

If *G.B. is a woman* **then** G.B. is a mother**.**

From the fact that G.B. is a woman it does not follow that she is a mother, though from the fact that G.B. is a mother it *does too* follow that she is a woman.

[2b] M → W
(2b) **If** G.B. is a mother, **then** [this must mean that] *G.B. is a woman*.

since only women can be mothers! Another way of expressing the same statement:

(2a) **If** *G.B. is* **not** *a woman* **then** G.B. is **not** a mother**.**
[2a] ~W → ~M

Again, both [2a] and [2b] are correct symbolizations of (2).

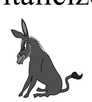
Example 3

One final example

(3) It rains **only if** *it is cloudy*.

C: It is cloudy
R: It rains

This statement is once again obviously true (the rain must fall from some cloud or other), but the sentence resulting from the natural mistake of assuming that the italicized sentence belongs in the antecedent is evidently false:



If *it is cloudy* **then** it rains**.**

From the fact that it is cloudy it does not follow that it rains. It can be cloudy and it can snow, it can be just plain cloudy with no precipitation at all. It does follow, however, from the fact that it rains that there must be some cloud or other in the sky:

(3b) **If** it rains, **then** [this must mean that] *it is cloudy*.
[3b] R → C

since only if it is cloudy can it rain! Another way of expressing the same statement:

(3a) **If** *it is* **not** *cloudy* **then** it does **not** rain**.**
[3a] ~C → ~R

Again, both [3a] and [3b] are permissible symbolizations of (3).

In general:

p if r	if r then p	$r \rightarrow p$
p only if r	if p then [this must mean that] r if not r then not p	$p \rightarrow r$ $\sim r \rightarrow \sim p$

Exercise Only If – 1

Offer two paraphrases of the following only-if conditionals and symbolize them:

(a) Trippy is a cat only if Trippy can meow.

1:

2:

C: Trippy is a cat
M: Trippy can meow

[1]

[2]

(b) Tramp is a dog only if Tramp can bark.

1:

2:

B: Tramp can bark
D: Tramp is a dog

[1]

[2]

(c) Truppy is a fish only if Truppy can swim.

1:

2:

F: Truppy is a fish
S: Truppy can swim

[1]

[2]

(d) It rains only if it is cloudy

1:

2:

C: It is cloudy
R: It rains

[1]

[2]

(e) It snows only if it is cloudy

1:

2:

C: It is cloudy
S: It snows

[1]

[2]

(e) It snows only if it is very cold.

1:

2:

C: It is very cold
S: It snows

[1]

[2]

(f) I will pass logic only if I work very hard.

1:

2:

P: I pass logic
W: I work very hard

[1]

[2]

Exercise Only If – 2

Provide the symbolizations of the following statements using the provided symbolization key. Provide two equivalent symbolizations for ‘only if’.

A: Ann is on a diet	E: Betty exercises
B: Betty is on a diet.	H: Ann is healthy
C: Charlie is on a diet	L: Betty is healthy

(a) Ann will be healthy only if she goes on a diet.

(b) Betty will be healthy only if either she goes on a diet or starts exercising regularly.

(c) Ann will go on a diet only if both Betty and Charlie go on a diet.

(d) Betty will either go on a diet or start exercising regularly only if Ann goes on a diet.

(e) Charlie will go on a diet, only if Betty goes on a diet but Ann does not.

(f) Betty will exercise only if she does not go on a diet.

(g) Only if Charlie is on a diet will Ann go on a diet.
Paraphrase:

(h) Only if Betty either is healthy or starts exercising will Charlie go on a diet.
Paraphrase:

(i) Ann and Betty will be healthy only if they both go on a diet.

Exercise Only-if – 3

Ascertain the truth or falsehood of the following claims:

(a) You will get an A for this course ***if*** you get 95% on all your quizzes. ❍true ❍false

(b) You will get an A for this course ***only if*** you get 95% on all your quizzes. ❍true ❍false

(c) You will get an A for this course ***if*** you work hard. ❍true ❍false

(d) You will get an A for this course ***only if*** you work hard. ❍true ❍false

3.5.3. Necessary and Sufficient Conditions

To grasp the difference between 'if' and 'only if' is to grasp the difference between sufficient and necessary conditions respectively. We express necessary conditions by means of 'only if'. Consider our examples:

> You will win the lottery **only if** *you buy the ticket*.
> NOT: You will win the lottery **if** *you buy the ticket*.

Buying the ticket is a necessary (though not sufficient) condition for winning a lottery. Buying the ticket is not a sufficient condition for winning a lottery because it is not the case that you will win the lottery if you buy the ticket.

> G.B. is a mother **only if** *G.B. is a woman*.
> NOT: G.B. is a mother **if** *G.B. is a woman*.

Being a woman is a necessary (again not a sufficient) condition for being a mother. Being a woman is not a sufficient condition for being a mother because it is not the case that if someone is a woman then someone is a mother (some women are not mothers).

> It rains **only if** *it is cloudy*.
> NOT: It rains **if** *it is cloudy*.

Being cloudy is a necessary (again not a sufficient) condition for rain. Being cloudy is not a sufficient condition for rain because it is no the case that it always rains if it is cloudy (sometimes there are clouds without precipitation, sometimes it snows when it is cloudy).

Consider some examples of sufficient (though not necessary) conditions.

> Ann will be angry **if** Stan again forgets about their anniversary.
> NOT: Ann will be angry **only if** Stan again forgets about their anniversary.

Stan's forgetting about the anniversary is a sufficient condition for Ann's getting angry: if Stan forgets about the anniversary then Ann will be angry. Stan's forgetting about the anniversary is not a necessary condition for Ann getting angry since it is the case that Ann will get angry *only* if Stan forgets about their anniversary, she might get angry for other reasons.

> It rains **if** *it pours*.
> NOT: It rains **only if** *it pours*.

Pouring is sufficient for raining since whenever it pours it rains. Pouring is not necessary for raining since it is not the case that it rains only if it pours – a light drizzle is still a rain.

> You will get an A **if** you score 92% on all your quizzes.
> NOT: You will get an A **only if** you score 92% on all your quizzes.

Scoring 92% on your quizzes is sufficient for your getting an A since if you score 92% you will get an A. Scoring 92% on your quizzes is not necessary condition for your getting an A since it is not the case that you will get an A only if you score 92% – you will get an A as long as you score more than 90%.

In general:

p if r	if r then p	r is a sufficient condition for p	$r \rightarrow p$
p only if r	if p then r [if not r then not p]	r is a necessary condition for p	$p \rightarrow r$ [$\sim r \rightarrow \sim p$]
p if and only if r		r is a necessary and a sufficient condition for p	$r \equiv p$

You must learn the symbolization schemata by heart!

3.6. ‘All’, ‘Some’, ‘Not all’, ‘None’

3.6.1. ‘All’ and ‘Some’

The words ‘all’ and ‘some’ indicate the so-called quantificational operators, which only appear, in all their glory, in predicate logic. Thus most of the occurrences of ‘all’ and ‘some’ cannot be expressed in terms of propositional logic. But some of those occurrences can be paraphrased in propositional logic.

For example, when there is a finite group of people that is being talked about, let us say: Ann, Betty, Charlie and Dirk. In such a case, the statement:

(1) All of them are on a diet.

will mean the same as:

(1′) Ann, Betty, Charlie and Dirk are on a diet.

which given the symbolization key:

A: Ann is on a diet   C: Charlie is on a diet
B: Betty is on a diet.   D: Dirk is on a diet

we can symbolize as any of the following:

[1a] (A • B) • (C • D)
[1b] A • ((B • C) • D)
[1c] A • (B • (C • D))
[1d] ((A • B) • C) • D
[1e] (A • (B • C)) • D

Similarly, when it is crystal clear exactly who is being talked about, we can understand a statement such as:

(2) At least one of them is on a diet.

or equivalently:

(2′) Some of them are on a diet.

to mean the same as:

(2″) Ann, Betty, Charlie or Dirk is on a diet.

which would be symbolized, for example (I’ll be skipping the parentheses permutations), as:

[2] (A ∨ B) ∨ (C ∨ D)

3.6.2. ‘None’ and ‘Not all’

Also “negative” quantificational expressions can be paraphrased in propositional logic provided that we restrict their range to a specified group of people. Consider the following statement (when it is clear that Ann, Betty, Charlie and Dirk are the people who can be meant):

(3) None of them is on a diet.

Statement (3) means the same as:

(3′) Neither Ann nor Betty nor Charlie nor Dirk is on a diet

which can be represented, for example (again skipping parentheses permutations), as:

[3a] $(\sim A \bullet \sim B) \bullet (\sim C \bullet \sim D)$

or equivalently as (bearing in mind two different ways of symbolizing “neither-nor”):

[3b] $\sim[(A \vee B) \vee (C \vee D)]$

We can also find a way of representing a statement like:

(4) None all of them are on a diet.

Statement (4) means the same as:

(4′) Ann, Betty, Charlie and Dirk are not all on a diet

which is best understood as a negation of the claim that they are all on a diet, i.e. as:

[4a] $\sim[(A \bullet B) \bullet (C \bullet D)]$

or equivalently as (bearing in mind two different ways of symbolizing “not-both”):

[4b] $(\sim A \vee \sim B) \vee (\sim C \vee \sim D)$

Exercise All-Some-None-Not-All

Provide the symbolizations of the following statements using the provided symbolization key, on the understanding the expressions 'all' and 'some' can pertain at most to the group of six persons: Ann, Betty, Charlie, Dirk, Evelyn and Frank.

A: Ann is on a diet	D: Dirk is on a diet
B: Betty is on a diet.	E: Evelyn is on a diet.
C: Charlie is on a diet	F: Frank is on a diet

(a) Ann, Betty and Charlie are all on a diet

(b) Dirk, Evelyn or Frank is on a diet.

(c) All six are on a diet

(d) At least one of the six is on a diet.

(e) All girls from this group are on a diet.

(f) Some boys from this group are on a diet.

(g) There is a girl in this group who is on a diet.

(h) Betty and Charlie are not both on a diet

(i) Dirk, Evelyn and Frank are not all on a diet.

(j) Neither Betty, Charlie nor Evelyn is on a diet.

(k) Not all girls in this group are on a diet

(l) No boys in this group are on a diet.

(m) Nobody in this group is on a diet.

(n) Not everybody in this group is on a diet.

4. Complicated Symbolizations

The material that you have now covered should give you enough preparation to face up to most symbolization tasks in propositional logic. You have to bear in mind that in the complicated symbolizations you will be required to apply all the "tricks" you have learned at once – this is what makes those symbolizations complicated. You should also bear in mind that very often there will be more than one correct way of symbolizing a statement. For example, if you get a statement that contains both 'only if' and 'neither nor' (supposing it contains no other connective-phrases) there will be four ways of symbolizing the statement. Let's do some examples together.

Example 1

(1) Jane will go out with Bill only if he neither smokes nor drinks heavily.

You have probably developed your eye sufficiently to know that we need three logical constants in the symbolization key:

D: Bill drinks heavily
J: Jane goes out with Bill
S: Bill smokes

Let's underline the connectives and place the parentheses – there is only one way to do so:

Jane will go out with Bill <u>only if</u> (he <u>neither</u> smokes <u>nor</u> drinks heavily)

It is now best to substitute the logical constants in:

J only if (neither S nor D)

And now you need to search back to your memory how "*p* only if *r*" is symbolized, and how "neither *p* nor *r*" is symbolized. There are two ways to symbolize "neither *p* nor *r*":

J only if (~S • ~D)
J only if ~(S ∨ D)

and two ways to symbolize 'only if'.

J → (neither S nor D)
~(neither S nor D) → ~J

There are thus at least four possible ways to symbolize (1):

[1a] J → (~S • ~D)
[1b] J → ~(S ∨ D)
[1c] ~(~S • ~D) → ~J
[1d] ~~(S ∨ D) → ~J

Example 2

(2) Either both Tim and Jim will make an A in logic or neither of them will.

This statement would benefit from rephrasing it:

> Either both Tim and Jim will make an A in logic or neither Tim nor Jim will make an A in logic.

We can now construct the symbolization key:

T: Tim will get an A in logic
J: Jim will get an A in logic

'Either-or' provides a good guide where to put the parentheses:

> Either (both Tim and Jim will make an A in logic) or (neither Tim nor Jim will make an A in logic).

Abbreviate the simple statements with logical constants:

> Either (T and J) or (neither T nor J)

We are ready to complete the process:

[2a] $(T \bullet J) \vee (\sim T \bullet \sim J)$
[2b] $(T \bullet J) \vee \sim (T \vee J)$

Example 3

Here is an example from V. Klenk's book:

(3) Neither stock prices nor consumer spending will fall, provided unemployment does not rise and there is a boom in either housing or the automobile industry.

This is a complicated statement. Let's begin by underlining the connectives and setting up a symbolization key:

<u>Neither</u> stock prices <u>nor</u> consumer spending will fall, <u>provided</u> unemployment does <u>not</u> rise <u>and</u> there is a boom in <u>either</u> housing <u>or</u> the automobile industry.

A: There is a boom in the automobile industry
F: Stock prices will fall
H: There is a boom in housing
S: Consumer spending will fall
U: Unemployment rises

The most important thing that we need to decide is what the logical structure of this statement is. Here both the comma and your intuitions concerning the meaning ought to tell you that the main connective is 'provided'. This is a conditional:

(<u>Neither</u> stock prices <u>nor</u> consumer spending will fall) <u>provided</u> (unemployment does <u>not</u> rise <u>and</u> there is a boom in <u>either</u> housing <u>or</u> the automobile industry)

Once you see this, the rest is easy. The main connective in the second parentheses is 'and':

(<u>Neither</u> stock prices <u>nor</u> consumer spending will fall) <u>provided</u> (unemployment does <u>not</u> rise <u>and</u> (there is a boom in <u>either</u> housing <u>or</u> the automobile industry))

Substituting logical constants:

(Neither F nor S) provided (not U and (either H or A))

We can now rephrase the 'provided' in the standard 'if…then…' form:

If (not U and (either H or A)) then (Neither F nor S)

The rest is a piece of cake:

[3a] $(\sim U \bullet (H \vee A)) \rightarrow (\sim F \bullet \sim S)$
[3b] $(\sim U \bullet (H \vee A)) \rightarrow \sim(F \vee S)$

Example 4

Here is another example from V. Klenk's book:

> (4) More jobs will be created and the economy will improve only if government spending is increased and taxes are not raised; however, the deficit will be reduced only if taxes are raised and government spending is not increased, and the economy will improve if and only if the deficit is reduced.

Again, let's underline the connectives and set up a symbolization key:

> More jobs will be created and the economy will improve only if government spending is increased and taxes are not raised; however, the deficit will be reduced only if taxes are raised and government spending is not increased, and the economy will improve if and only if the deficit is reduced.

D: Deficit is reduced
E: The economy improves
G: Government spending increases
J: More jobs are created
T: Taxes are raised

Here, once again, the crucial thing is to find the main connective. It is relatively easy to do so. The big indication of a "break" in the sentence is provided by the semicolon and the word 'however'. (If you read the sentence out loud, this is where you will pause.)

> (More jobs will be created and the economy will improve only if government spending is increased and taxes are not raised); however, (the deficit will be reduced only if taxes are raised and government spending is not increased, and the economy will improve if and only if the deficit is reduced).

But now it turns out that the parentheses contain complex statements themselves and we need to find the main connectives for those statements. In the case of:

> More jobs will be created and the economy will improve only if government spending is increased and taxes are not raised.

the most natural way of reading the statement is to treat it as a conditional, where 'only if' is the main connective. Here the meaning of what is said is our guide. If any of the other connectives were intended as the main ones, that would have to be marked by punctuation or additional clauses such as 'in any event'.
In the case of the statement in the second parentheses:

> The deficit will be reduced only if taxes are raised and government spending is not increased, and the economy will improve if and only if the deficit is reduced.

the comma indicates that the second ‘and’ is the main connective. The statement is a conjunction of a conditional and a biconditional. We thus have:

> ((More jobs will be created <u>and</u> the economy will improve) <u>only if</u> (government spending is increased <u>and</u> taxes are <u>not</u> raised))<u>; however,</u> ((the deficit will be reduced <u>only if</u> (taxes are raised <u>and</u> government spending is <u>not</u> increased))<u>, and</u> (the economy will improve <u>if and only if</u> the deficit is reduced))

Substituting logical constants:

> ((J and E) only if (G and not T)); however, ((D only if (T and ~G)), and (E if and only if D))

The rest is easy, though remember that there are two ways to symbolize ‘only if’ (I provide the simplest one – without the negation):

[4] $((J \bullet E) \rightarrow (G \bullet {\sim}T)) \bullet ((D \rightarrow (T \bullet {\sim}G)) \bullet (E \equiv D))$

Exercise Symbolizations – 4

Please symbolize the following statements, using the symbolization key provided.

A: Ann is on a diet	L: Larry is getting fat
B: Betty is on a diet.	M: Martin is getting fat
C: Charlie is on a diet	N: Newt is getting fat

(a) Either both Ann and Betty are on a diet, or neither of them is.

(b) Either both Ann and Betty are on a diet, or not both of them are.

(c) Not both Larry and Martin are getting fat, though both Martin and Newt are getting fat.

(d) It is both the case that neither Ann is on a diet nor Larry is getting fat and that neither Betty nor Charlie are on a diet.

(e) Either neither Ann nor Charlie is on a diet or neither Betty nor Charlie is on a diet.

(f) It is not the case that neither Ann nor Charlie is on a diet.

(g) It is not the case that both Martin and Newt are getting fat

(h) It is not the case that not both Martin and Newt are getting fat.

(i) It is not both the case that neither Ann nor Betty is on a diet and that neither Betty nor Charlie is on a diet.

Hint: Symbolize first the statement that is being negated here, i.e. the statement that is enclosed in parentheses:
It is not [both the case that neither Ann nor Betty is on a diet and that neither Betty nor Charlie is on a diet].

Exercise Symbolizations – 5

Symbolize the following statements, using the first letters of a name to abbreviate the simple statements, e.g.:

A: Amy is nice
B: Betty is nice
…

(a) Both Jennifer and Katrina are nice but Susan certainly is not.

(b) Neither Amy nor Susan are nice, though Mary is very nice.

(c) Katrina and Mary are never both nice.

(d) Katrina and Susan are never both nice, either.

(e) Either Jennifer is nice or Mary is nice, but never both.

(f) Jennifer is nice just in case Mary or Katrina are nice.

(g) If Jennifer is nice, then Lucy is nice provided that Donna is nice.

(h) Lucy is nice if Jennifer is nice, provided that Donna is nice.

(i) Donna is not nice unless Jennifer is nice.

(j) Mary is nice unless Amy and Betty are both nice.

(k) Susan is not nice unless either Amy or Betty are nice

(l) Jennifer, Katrina, Mary and Lucy are all nice.

(m) At least one of these four girls is nice.

Exercise Symbolizations – 6 (after V. Klenk)

Symbolize the following statements, using the first letters of a name to abbreviate the simple statements, e.g.:

A: There is a boom in the automobile industry
C: Consumers increase borrowing
D: Deficit is reduced
E: The economy improves
F: Stock prices will fall
G: Government spending increases
H: There is a boom in housing
J: More jobs are created
R: Interest rates rise
S: Consumer spending will fall
T: Taxes are raised
U: Unemployment rises

(a) Interest rates will rise only if the economy improves and consumers increase borrowing.	
(b) The economy will not improve and interest rates will not rise if either consumer spending falls or unemployment rises.	
(c) Either interest rates or unemployment rates will rise, but not both.	
(d) Interest rates will not rise if the economy improves, provided consumers do not increase borrowing.	
(e) The deficit will be reduced and the economy will improve if taxes are raised and interests rates do not rise	
(f) The deficit will be reduced if and only if taxes are raised and government spending does not increase, unless interest rates rise.	
(g) Unless the deficit is reduced, taxes and interest rates will rise and the economy will not improve.	
(h) Stock prices will fall and the economy will fail to improve if interest rates rise and the deficit is not reduced, unless either more jobs are created or there is a boom in housing.	
(i) Neither taxes nor interest rates will rise if the deficit is reduced, but if the deficit is not reduced then both taxes and interest rates will rise.	

(j) The economy will improve if the deficit is reduced, but the deficit will be reduced only if government spending does not increase and taxes are raised.

(k) Stock prices will fall and either interest rates or unemployment will rise, unless either the deficit is reduced and the economy improves or taxes are not raised and consumer spending does not fall.

(l) Only if there is a boom in housing and the automobile industry will more jobs be created and the deficit be reduced, but more jobs will not be created unless government spending increases.

A: There is a boom in the automobile industry
C: Consumers increase borrowing
D: Deficit is reduced
E: The economy improves
F: Stock prices will fall
G: Government spending increases

H: There is a boom in housing
J: More jobs are created
R: Interest rates rise
S: Consumer spending will fall
T: Taxes are raised
U: Unemployment rises

5. Tricky Symbolizations

5.1. Not Everything that Looks Like a Conjunction Is a Conjunction

You already know that symbolization need not be so straightforward. Here is one more complication in the case of conjunction. Consider first a straightforward case:

(1) Susan and Mary wear glasses.

This is a straightforward conjunction since it can be rendered more boringly but more fully as a conjunction of two simple statements:

(1′) Susan wears glasses and Mary wears glasses.

The logical structure of this statement is thus:

[1] S • M

where

S: Susan wears glasses
M: Mary wears glasses

The same is true for many of the occurrences of ‘and’.

There are some uses of ‘and’ where a conjunction is not even in sight. Consider:

(2) Susan and Will are related.

Surely, we cannot interpret this statement as a conjunction:

Susan is related *and* Will is related.

We do not even understand what such a statement means, but it certainly does not mean what we originally said, viz. that Susan and Will were related. Rather, the original statement ought to be understood thus:

Susan is related to Will.

There is no conjunction here. This is a simple statement (from the point of view of propositional logic). Given the symbolization key:

S: Susan is related to Will

we can represent (2) as:

[2] S

There are other examples where ‘and’ does not function as a conjunction. Consider:

(3) Jack and Jill are married
(4) Fichte and Hegel were contemporaries.

and so on.

5.2. ‘Not Only … But…’

Consider the statement:

(1) Ann is not only a loving mother but also a dedicated scientist.

The presence of ‘not’ might suggest that some negation is in sight. The presence of ‘only’ might wake you up. In fact, however, when you reflect on what is being said, you will see that what the person means to say (from the point of view of propositional logic) is just:

(1′) Ann is both a loving mother and a dedicated scientist.

This is simply a conjunction:

[1] L • D

where:

L: Ann is a loving mother
D: Ann is a dedicated scientist

6. Summary

In this unit, you should have acquired the skill of symbolizing even very complicated statements. It is important to remember the symbolization "tricks" you have learned.

7. What You Need to Know and Do

- You need be able to symbolize statements, both less and more complicated.
- You need be able to symbolize some statements in two ways, this includes the symbolization of statements containing the following connective-phrases: 'neither-nor', 'not-both' (as well as 'none' and 'not all'), 'only-if', 'unless', 'either-or-but-not-both'.