Solutions to Workbook Exercises

Unit 2:

The Basics of Propositional Logic

Exercise "Simple vs. Complex Propositions"

Which of the following propositions are simple? Which are complex? In case of complex propositions underline the connectives by means of which the proposition is constructed.

Simple sentences are highlighted

(a)   Tom is one of those extraordinarily nice men who hate all women that wear big hats.

(b)   Tom invited Susan out **and** she agreed.

(c)   **If** Susan does **not** come on time**,** Tom will be distraught.

(d)   Susan was quite punctual.

(e)   Tom did **not** believe his eyes.

(f)   Susan was wearing the biggest hat Tom has ever seen in his entire life.

(g)   Tom starting pleading for Susan to take off what seemed to him to be one of the ugliest things ever produced by a human hand.

(h)   Susan agreed to take off the hat **if and only if** Tom takes off the bow-tie **and** cowboy boots.

Exercise "Symbolization Key"

(a)   Tom invited Susan out and she agreed to go out with him.

(b)   If Susan agrees to go out with Tom then she will wear a big hat.

(c)   Susan will wear a big hat if and only if she will want to teach Tom a lesson.

(d)   If Susan wants to teach Tom a lesson, then he will not invite her out.

Symbolization key (any four different letters could be used)**:**

T:	Tom invites Susan out.
A:	Susan agrees to go out with Tom.
H:	Susan wears a big hat.

> **L:**   Susan wants to teach Tom a lesson.

Exercise "Negations"

Symbolize the following statements using the symbolization key provided.

> A: Abe will make dinner
> B: Betty will make dinner

(a)	Abe will not make dinner	~A
(b)	It would be false to say that Betty will make dinner.	~B
(c)	It would be false to say that Betty will not make dinner.	~~B
(d)	It would be preposterous to think that Abe will make dinner.	~A
(e)	It would be preposterous to think that it would not be the case that Betty will not make dinner.	~~~B
(f)	Abe failed to make dinner	~A

Exercise "Conjunctions"

Symbolize the following statements using the symbolization key provided.

> A: Abe will make dinner
> B: Betty will make dinner
> C: Chris will make lunch

(a)	Abe and Betty will both make dinner.	A • B
(b)	Abe will make dinner though Betty will not.	A • ~B
(c)	Abe will not make dinner even though Betty will.	~A • B
(d)	Abe and Betty will make dinner while Chris will make lunch.	(A • B) • C
(e)	Chris will not make lunch; moreover, Abe will not make dinner	~C • ~A
(f)	Despite the fact that Abe will not make dinner, Betty will make it. *Paraphrased:* Betty will make dinner despite the fact that Abe will not make dinner.	B • ~A[1]
(g)	Abe will not make dinner but still Betty will make it.	~A • B

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[1] B • ~A is logically equivalent to ~A • B. Both symbolizations are fine.

Exercise "Disjunctions"

Symbolize the following statements using the symbolization key provided.

A: Abe will make dinner
B: Betty will make dinner
C: Chris will make lunch
D: Dan will make lunch

(a)	Abe or Betty will make dinner.	$A \vee B$
(b)	Either Abe or Betty will make dinner.	$A \vee B$
(c)	Chris will make lunch while either Abe or Betty will make dinner.	$C \bullet (A \vee B)$
(d)	Either Chris or Dan will make lunch.	$C \vee D$
(e)	Either Chris will make lunch or Betty will not make dinner.	$C \vee {\sim}B$
(f)	Either Chris will not make lunch or Dan will not make lunch.	${\sim}C \vee {\sim}D$
(g)	Either Chris or Dan will make lunch and either Abe or Betty will make dinner.	$(C \vee D) \bullet (A \vee B)$
(h)	Abe and Betty will make dinner.	$A \bullet B$
(i)	Either Abe and Betty will make dinner or Chris and Dan will make lunch.	$(A \bullet B) \vee (C \bullet D)$
(j)	Either Abe will make dinner while Chris will make lunch or Betty will make dinner while Dan will make lunch.	$(A \bullet C) \vee (B \bullet D)$
(k)	Betty will make dinner and either Chris or Dan will make lunch.	$B \bullet (C \vee D)$
(l)	Either Abe will make dinner or either Chris or Dan will make lunch.	$A \vee (C \vee D)$

Exercise "Biconditionals"

Symbolize the following statements using the symbolization key provided.

A: Abe will make dinner
B: Betty will make dinner
C: Chris will make lunch

(a) Abe will make dinner if and only if Betty will.

$$A \equiv B$$

(b) Chris will make lunch just in case Betty makes dinner.

$$C \equiv B$$

(c) Abe makes dinner when only when Betty does not.

$$A \equiv \sim B$$

(d) Chris will make lunch just in case Abe or Betty make dinner.

$$C \equiv (A \lor B)$$

Exercise "Conditionals 1"

Symbolize the following statements using the symbolization key provided.

A: Abe will make dinner
B: Betty will make dinner
C: Chris will make lunch
D: Dan will make lunch

(a) If Chris makes lunch then Betty will make dinner.

$$C \to B$$

(b) Chris will make lunch if Dan does not make lunch.

If	Dan does not make lunch
then	Chris will make lunch

$$\sim D \to C$$

(c) Dan will not make lunch if Abe does not make dinner.

If	Abe does not make dinner.
then	Dan will not make lunch.

$$\sim A \to \sim D$$

(d)   Dan will make lunch if either Abe or Betty make dinner.

If	either Abe or Betty make dinner
then	Dan will make lunch

$(A \lor B) \rightarrow D$

(e)   On the condition that Betty makes dinner, Chris will make lunch.

If	Betty makes dinner
then	Chris will make lunch

$B \rightarrow C$

(f)   Betty will make dinner on the condition that Chris makes lunch.

If	Chris makes lunch
then	Betty will make dinner

$C \rightarrow B$

(g)   Abe will make dinner on the assumption that Betty does not.

If	Betty does not make dinner
then	Abe will make dinner

$\sim B \rightarrow A$

(h)   Assuming that Chris does not make lunch, Dan will make it.

If	Chris does not make lunch
then	Dan will make lunch

$\sim C \rightarrow D$

(i)   Abe will make dinner provided that either Chris or Dan make lunch.

If	either Chris or Dan make lunch
then	Abe will make dinner

$(C \lor D) \rightarrow A$

(j) Provided that Abe will make dinner, Betty will not make it.

If	Abe will make dinner
then	Betty will not make dinner

$A \rightarrow \sim B$

(k) As long as Abe makes dinner, Betty will not make it.

If	Abe makes dinner
then	Betty will not make dinner.

$A \rightarrow \sim B$

(l) Given that either Chris or Dan make lunch, Abe or Betty will make dinner.

If	either Chris or Dan make lunch
then	Abe or Betty will make dinner

$(C \lor D) \rightarrow (A \lor B)$

(m) Chris and Dan will both make lunch given that Abe and Betty both make dinner.

If	Abe and Betty both make dinner
then	Chris and Dan will both make lunch

$(A \bullet B) \rightarrow (C \bullet D)$

(n) It would be false to claim that if Betty makes dinner then Abe will make dinner.

$\sim(B \rightarrow A)$

(o) It would be false to claim that if Betty does not make dinner then Abe will not make dinner.

$\sim(\sim B \rightarrow \sim A)$

Exercise "Conditionals 2"

Please symbolize the following sentences, using the following symbolization key. Paraphrase the sentences where indicated into the "if… then…" form.

D: Ann is on a <u>d</u>iet
E: Ann <u>e</u>xercises regularly
F: Ann is getting <u>f</u>at
H: Ann is getting <u>h</u>ealthier
I: Billy is on a d<u>i</u>et
J: Billy <u>j</u>ogs regularly
T: Billy is getting fa<u>t</u>

(a) If Ann exercises regularly, she will get healthier.

$E \rightarrow H$

(b) Ann will get healthier if she goes on a diet.

If	Ann goes on a diet
then	Ann will get healthier

$D \rightarrow H$

(c) Ann will go on a diet if Billy goes on a diet

If	Billy goes on a diet
then	Ann will go on a diet

$I \rightarrow D$

(d) Billy will go on a diet provided that Ann goes on a diet

If	Ann goes on a diet
then	Billy will go on a diet

$D \rightarrow I$

(e) Given that Billy jogs regularly, he is not getting fat.

If	Billy jogs regularly
then	Billy is not getting fat

$J \rightarrow \sim T$

(f) Ann will exercise regularly on the condition that Billy jogs regularly.

If	Billy jogs regularly
then	Ann will exercise regularly

$J \rightarrow E$

(g) Provided that Ann is not getting fat, she will be getting healthier.

If	Ann is not getting fat
then	Ann will be getting healthier

$\sim F \rightarrow H$

(h) Ann exercises regularly given that she is getting healthier.

If	Ann is getting healthier
then	Ann exercises regularly

$H \rightarrow E$

(i) On the condition that Billy jogs regularly, Ann will be exercising regularly as well.

If	Billy jogs regularly
then	Ann will be exercising regularly as well

$J \rightarrow E$

Exercise "Main Connective 1"

Using the method of parentheses binding, find the main connective in each of the following propositions:

1. $(A \cdot B) \vee (C \rightarrow D)$

5. $(A \cdot (B \vee C)) \rightarrow D$

2. $A \cdot (B \vee (C \rightarrow D))$

6. $(A \rightarrow A) \rightarrow (A \rightarrow B)$

3. $((A \cdot B) \vee C) \rightarrow D$

7. $A \rightarrow (A \rightarrow (A \rightarrow B))$

4. $A \cdot ((B \vee C) \rightarrow D)$

8. $((A \rightarrow A) \rightarrow A) \rightarrow B$

9. $(((A \cdot B) \cdot C) \equiv (A \vee C)) \rightarrow (A \cdot (B \vee C))$

10. $(((A \equiv B) \rightarrow (B \equiv C)) \cdot (C \rightarrow D)) \vee (B \rightarrow ((A \cdot B) \equiv C))$

11. $(((A \vee B) \cdot (C \vee D)) \rightarrow C) \equiv ((A \rightarrow ((C \rightarrow D) \rightarrow B)) \rightarrow D)$

Exercise "Main Connective 2"

Using the method of parentheses binding, find the main connective in each of the following propositions:

1. ~A → (B ∨ A)

5. ~(~A ∨ B) → C

2. ~(A → B) ∨ A

6. ~(~A ∨ (B → C))

3. ~((A → B) ∨ A)

7. ~~(A ∨ (B → C))

4. ~~A ∨ (B → C)

8. ~~(A ∨ B) → C

9. ~(~(~A ∨ ~B) → ~(A • B)) → ~(~A ∨ ~A)

10. ~(~(A → (B • A)) ≡ ~~~(~B → ~A))

11. ~~(~A ∨ ~(B • ~C)) → ~((~C ∨ B) ≡ A)