Workbook Unit 2:

The Basics of Propositional Logic

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Overview

In this unit you will be introduced to the basics of an old logical theory, the so-called propositional or statement logic. We will discuss the five basic connectives that are at the center of the theory.

If you found the first unit easy, this might not be the case for the second. Start right away. It is also important for you to follow the reading and the exercises in the order I suggest. It is very important that you do them.

These early sections train your hand and your eye. – Only by doing the exercises will you be able to acquire the right kinds of skills. It is easy to get the impression that these exercises are too easy to bother and that you really do not gain anything by doing them. This is wrong! Even if they do come easy to you, they actually instill in you multiple skills that will be of enormous importance to you later. And given the deadline structure, there is no turning back. – Remember what I told you at the beginning. There is 0.001% of people who do not need to do the exercises – their mind already works formally. For most of us (myself included), all these exercises are a must!

This unit

- introduces the distinction between simple and complex propositions
- introduces five connectives (of negation, conjunction, disjunction, conditional, biconditional) by means of which complex propositions may be formed, and thus:
- introduces the basic truth tables for the connectives
- introduces the symbols for the connectives
- teaches how to symbolize propositions that are not very complicated
- introduces the idea of the main connective
- tells you how to determine the main connective of even very complicated propositions

PowerPoint Presentation

There is a PowerPoint Presentation that accompanies this Unit. It is available on-line as a .pps and a zipped .pps file.

On-Line Exercises

There are some on-line exercises for this unit that will help you in grasping some of the concepts (e.g. the idea of the main connective) and in developing the skill of symbolizations of conditionals.

PropositionalLogic Quiz

Check the Calendar for the deadlines for PropositionalLogic Quiz #1 and PropositionalLogic #2. See the Quiz Instructions and the Sample Quiz.

1. Propositional Logic as a Logical Theory

One of the goals of the science of logic is to understand what arguments are valid. This turns out to be a very difficult task and logicians have approached it step-by-step by first proposing relatively simple theories, and only later theories that gradually become more and more complex and also better at capturing what we intuitively consider to be valid arguments.

The first and the simplest of such theories is called propositional logic or statement logic (or sometimes even sentential logic). It is a relatively simple theory (as logical theories go), which is not to say that all the logical techniques that it introduces are likewise simple. One of the basic assumptions of this theory concerns the question of what the logical structure of propositions is. According to propositional logic, propositions are made up of other simpler propositions by means of the so-called propositional connectives (or connectives, for short), i.e. such expressions as ‘not’, ‘and’, ‘or’, ‘if…then’ and ‘if and only if’).

Propositional logic is quite impressive given its simplicity. But it is important to remember that it is a simple theory and it also has its problems. Some of its concepts do not capture our intuitions adequately, though others do. I will be stressing the points at which the theory is oversimplifying our complex intuitions. Be prepared for that. There are other theories that deal with such (and other) problems better than propositional logic.

One of such better, but also more complex, theories is the so-called quantificational logic (which includes predicate logic and the logic of relations). It captures as valid all those arguments that turn out to be valid in propositional logic but it also captures as valid some other arguments that propositional logic wrongly qualifies as invalid. This is because quantificational logic has a more refined way of analyzing the logical structure of complex propositions. It treats complex propositions as not only composed of the connectives but also as composed of the so-called quantifiers (expressions such as ‘all’ and ‘some’).

But this is not the end of the story. It turns out that quantificational logic is not the ultimate theory, for it again does not capture as valid all the arguments that we would want it to capture as valid. And so modal logics have been proposed, which analyze the behavior of such operators as ‘necessarily’ and ‘possibly’. Deontic logics analyze the behavior of ‘right’ and ‘duty’. And so on, and so forth.

The science of logic is very much an open-ended science. You will be just getting your feet wet in this class.

2. Simple and Complex Propositions

We have said in the last unit that arguments are valid (or invalid) in virtue of their logical form. It is primary task of a logical theory to tell us what logical form is. And it is here that the distinction between simple propositions and complex propositions comes in.

Complex propositions (in propositional logic) are those that are composed out of other propositions by means of any of the five connectives: *not*, *and*, *or*, *if…then*, *if* *and only if*. Propositions not so composed are called simple propositions. Consider some examples of complex propositions first (the occurrences of the connectives have been underlined):

> Charlie did **not** ask what the time is.
> Susan borrowed a book from Ann **and** she did **not** return it.
> **If** Alan keeps his room tidy for more than a week **then** he will get a cat **or** a dog.
> Kerstin will go out with Tim **if and only if** he apologizes to her first.

The following propositions are simple (in propositional logic):

> The Speaker of the House has confirmed the rumor that the work on the health bill has stopped three weeks before Christmas.
> Mary will feed the hamster
> All men are jealous.
> Whoever lives around the corner of Elm Street has a horrible view.

Note that simple propositions need not be expressed by simple sentences. What accounts for logical simplicity is the lack of connectives in a proposition, or more precisely the lack of a complex logical structure of being built out of simpler propositions by means of connectives.

Definition of complex and simple propositions

A complex proposition (in propositional logic) is a proposition constructed from other propositions by means of connectives.

A simple proposition (in propositional logic) is a proposition that is not constructed from other propositions by means of connectives.

Exercise "Simple vs. Complex Propositions"

Which of the following propositions are simple? Which are complex? In case of complex propositions underline the connectives by means of which the proposition is constructed.

(a) Tom is an extraordinarily nice man who hates all women that wear big hats.

(b) Tom invited Susan out and she agreed.

(c) If Susan does not come on time, Tom will be distraught.

(d) Susan was quite punctual.

(e) Tom did not believe his eyes.

(f) Susan was wearing the biggest hat Tom has ever seen in his entire life.

(g) Tom starting pleading for Susan to take off that awful thing, which must have been one of the ugliest things ever produced by a human hand.

(h) Susan agreed to take off the hat if and only if Tom takes off the bow-tie and cowboy boots.

3. The Symbolization Key (Legend)

One of the important things that we will be doing is called “symbolization.” We will be representing propositions expressed by means of English sentences using the symbolic notation of propositional logic. The first thing that you should know is that simple propositions are represented by the so-called *propositional constants*, which will always be the capital letters of the alphabet (A, B, C, etc.).

What letters stand for what proposition is completely arbitrary, though it is useful to select a letter that will clearly remind us of the proposition it represents. When we are to choose the letters to stand for the following propositions:

Tom is handsome.
Tom is rich.
Susan is pretty.
Susan is poor.

it would be most useful to use the first letters of the names of the characteristics that are being attributed to Tom and Susan. One possible assignment would be this:

H: Tom is handsome.
R: Tom is rich.
P: Susan is pretty.
O: Susan is poor.

But other assignments are also possible (you could use T, O, S, U, respectively, or even A, B, C, D). Such a list of the assignments of propositional constants (we will also speak of letters, for short) to simple propositions is called the *symbolization legend* or the *symbolization key*.

While the assignment of letters to simple propositions is arbitrary, three rules must be obeyed:

- *One proposition cannot be represented by more than one letter.*
  This rule is actually not so difficult to break. For we must remember that the same proposition can be expressed by many sentences. Consider, for example, the following sentences:

R: Tom is rich
M: Tom is a rich man.
S: Tom is a rich person.

Although different sentences are used, they all express the same proposition and should be represented by a single letter. It is thus enough to have one entry in the symbolization key, e.g.:

R: Tom is rich

- *One letter cannot represent more than one proposition.*
  We have followed this second rule in assigning different letters to the last two propositions above. We would have failed to follow this rule if we assigned the letter ‘P’ thus:

P: Susan is pretty.
P: Susan is poor.

- *No letter can represent a complex proposition*
  One final point to remember is that the symbolization key is not to include any complex propositions. We will need to represent the logical structure of complex propositions by means of logical symbols. In this way, we will be exhibiting their logical form.

I: If Tom is rich then he is not handsome.
N: Tom is not a rich man.

In this case, the following symbolization key would serve its purpose:

H: Tom is handsome.
R: Tom is rich

Exercise "Symbolization Key"

Construct a symbolization key for the following set of propositions, making sure that you obey all three rules for constructing symbolization keys. Hint: You should begin by listing all the simple propositions there are in all four propositions (a)-(d). Remember that sometimes different sentences express the same proposition. Note also that

➲ in propositional logic, we usually disregard the temporal indices of sentences. We will take the sentences 'Tom invites Susan out', 'Tom invited Susan out' and 'Tom will invite Susan out' to express the same proposition.

This is a simplification, which has to do with the fact that we do not index the sentences spatiotemporally, which would be most tedious to do.

(a) Tom invited Susan out and she agreed to go out with him.

(b) If Susan agrees to go out with Tom then she will wear a big hat.

(c) Susan will wear a big hat if and only if she will want to teach Tom a lesson.

(d) If Susan wants to teach Tom a lesson, then he will not invite her out.

Symbolization key:

:	Tom invites Susan out.
:	Susan agrees to go out with Tom.
:	Susan wears a big hat.
:	Susan wants to teach Tom a lesson.

4. Five Types of Complex Propositions

We will be now concerned to learn more about five types of complex propositions (negations, conjunctions, disjunctions, biconditionals and conditionals), which are constructed from simpler propositions by means of the five connectives.

4.1. Negation

Connective:	It is not the case that
Symbol:	~ [the tilde]
Logical form:	~ ☐
	~*p*
Component:	*p* – negated proposition

(Note that any proposition can enter into the box ☐)

Here is an example of a negation:

(1) **<u>It is not the case that</u>** the sun shines.

A negation is a complex proposition, which is composed of the negation connective (a tilde) and some proposition (which can be simple or complex), which is called the negated proposition. In sentence (1), the negated proposition is the proposition "The sun shines".

The negation connective is called a one-place connective because it forms a complex proposition out of *one* simpler proposition. All the other connectives we will be learning about are two-place connectives – they form a complex proposition out of two simpler propositions.

The Basic Truth Table for Negation

The meaning of the negation connective is given by the so-called basic truth table for negation. (Logicians will say that the basic truth table specifies the semantic properties of the negation connective.) In layman's terms, a truth table for negation tells us how the truth-value of a negation depends on the truth-values of the negated proposition.

In the case of negation, the truth table is exceedingly intuitive. The negated proposition can be either true or false. If the negated proposition *p* is true then the negation ~*p* will be (of course) false. If the negated proposition *p* is false then the negation ~*p* will be (of course) true.

Check that this is so on the following example. Consider the proposition "The cup is full of coffee". Let us first consider a situation when the proposition "The cup is full of coffee" is true (The capital letter 'T' will abbreviate 'true', while the capital letter 'F' will abbreviate 'false').

	p	*~p*
	The cup is full of coffee	The cup is not full of coffee
	T	

Given that this is the way things are (the cup is in fact full of coffee), when someone says “The cup is *not* full of coffee,” he or she is saying something false. (Write in ‘F’ in the above space.)

If, on the other hand, the proposition “The cup is full of coffee” is as a matter of fact false, as it is in this situation:

	p	*~p*
	The cup is full of coffee	The cup is not full of coffee
	F	

then when someone says “The cup is *not* full of coffee”, he or she is saying something true. (Write in ‘T’ in the above space.)

This relationship between the truth-value of a negated proposition and the resulting truth-value of the negation will always be preserved, no matter what propositions we consider. This is summarized in the basic truth table for negation thus:

Truth-value of negation:

p	*~p*
T	
F	

Fill in the truth-values. (You should check at the end of this unit that you have done so correctly.)

Negations in English

The expression ‘it is not the case that’ is a paradigmatic reading of the negation connective. However, negations can be expressed in English by means of many other phrases as well. The following sentences express the very same proposition as that expressed by sentence (1):

(2) The sun does **not** shine.
(3) **It would be false to say that** the sun shines.
(4) **The claim that** the sun shines **is a lie.**
(5) The sun **failed** to shine.

While these sentences do differ in meaning, they all express the same proposition from the point of view of propositional logic for the truth-value of all these complex propositions depends on the truth-value of the component proposition in the way characteristic of negation.

In order to symbolize these sentences, we need to provide a symbolization key:

S: The sun shines.

We can now symbolize (1)-(5) as:

[1] ~S

In the above case, the negated proposition was the simple proposition "The sun shines." However the negated proposition may be complex. Consider the following sentence:

(6) **<u>It is not the case that</u>** the sun does **<u>not</u>** shine.

This is a negation of a negation, which we can represent as follows:

[6] ~~S

Of course, there is no reason to stop there. One can negate even more complex sentences:

(7) **<u>It would be a lie to claim that</u>** **<u>it is not the case that</u>** the sun does **<u>not</u>** shine.

which would be symbolized as:

[7] ~~~S

At this point, you might wonder whether we could not represent (6) by means of proposition [1], i.e. as ~S. In fact, propositions ~~~S and ~S are logically equivalent. It is a general rule, however, that

➲ symbolizations ought to preserve the logical structure of the original proposition as closely as possible.

One reason why this is a good rule is that often times we are in fact confused about what is logically equivalent to what. We need logic and its techniques to help us.

Exercise "Negations"

Symbolize the following statements using the symbolization key provided.

A: Abe will make dinner
B: Betty will make dinner

(a)	Abe will not make dinner.	
(b)	It would be false to say that Betty will make dinner.	
(c)	It would be false to say that Betty will not make dinner.	
(d)	It would be preposterous to think that Abe will make dinner.	
(e)	It would be preposterous to think that it would not be the case that Betty will not make dinner.	
(f)	Abe failed to make dinner.	

4.2. Conjunction

Connective:	and
Symbol:	• [the dot]
Logical form:	□ • □
	$p \bullet q$
Components:	*p, q* – conjuncts

Here is an example of a conjunction:

(1) Ann has an apple **and** Ann has a banana.

For stylistic reasons we would rather say "Ann has an apple and a banana", which expresses the very same proposition that sentence (1) expresses. However, sentence (1) makes it more clear that conjunction is a connective that binds two propositions – a two-place connective.

The components of a conjunction are called conjuncts. In (1), the first conjunct is the proposition "Ann has an apple", while "Ann has a banana" is the second conjunct.

The Basic Truth Table for Conjunction

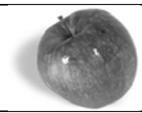
The basic truth table for conjunction is also very intuitive. The truth table tells us how the truth-value of a conjunction $p \bullet q$ depends on the truth-values of the conjuncts *p* and *q*. Since we have two component propositions (*p* and *q*) we now need to consider all possible truth-value assignments for *p* and *q*. There are four different possibilities:

p is true, *q* is true
p is true, *q* is false
p is false, *q* is true
p is false, *q* is false

If at this point you know how to fill out the truth table for conjunction you may skip the more detailed explanation.

	p	*q*	$p \bullet q$
[What Ann has:]	Ann has an apple	Ann has a banana	Ann has an apple and a banana
	T	T	

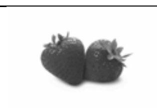
If it is true that Ann has an apple and it is also true that Ann has a banana, then it will be true to say that Ann has an apple and a banana.

[What Ann has:]	*p*	*q*	*p* • *q*
	Ann has an apple	Ann has a banana	Ann has an apple and a banana
	T	F	

If it is true that Ann has an apple but it is not true that Ann has a banana, then it will be false to say that Ann has an apple and a banana. Likewise:

[What Ann has:]	*p*	*q*	*p* • *q*
	Ann has an apple	Ann has a banana	Ann has an apple and a banana
	F	T	

If it is not true that Ann has an apple but it is true that Ann has a banana, then it will be false to say that Ann has an apple and a banana. Finally:

[What Ann has:]	*p*	*q*	*p* • *q*
	Ann has an apple	Ann has a banana	Ann has an apple and a banana
	F	F	

If it is not true that Ann has an apple and it is not true that Ann has a banana, then it will be false to say that Ann has an apple and a banana.

> A conjunction is true if and only if *both* conjuncts are true. A conjunction is false if and only if any conjunct is false.

Fill in the truth-values. (You should check at the end of this unit that you have done so correctly.)

	p	*q*	*p* • *q*
Truth-value of conjunction:	T	T	
	T	F	
	F	T	
	F	F	

Conjunctions in English

There are in fact a lot of ways in which conjunctions may be expressed in English, among others:

... and ...
both ... and ...
... as well as ...
... but ...
... however ...
... though ...
... although ...
... even though ...
... nevertheless …
... still …
... but still …
... also …
... while …
... despite the fact that …
... moreover …
... in addition …
...; ...
..., ...

The subtle meaning variations are disregarded by the theory of propositional logic which treats as a conjunction any sentence whose truth-value depends on the truth-values of the conjuncts in the way specified by the basic truth table for conjunction.

Let us consider some examples:

(2) John loves Mary **<u>even though</u>** she barely tolerates him.

Given the symbolization key:

J: John loves Mary
M: Mary barely tolerates John

we can symbolize sentence (2) as:

[2] J • M

From the point of view of propositional logic, the conjunctive proposition [2] also represents the following sentences:

(2) John loves Mary **<u>despite the fact that</u>** she barely tolerates him.
(3) John loves Mary **<u>and</u>** she barely tolerates him.
(4) John loves Mary **<u>but</u>** she barely tolerates him.
(5) John loves Mary **<u>while</u>** she barely tolerates him.
(6) John loves Mary **<u>nevertheless</u>** she barely tolerates him.
(7) John loves Mary **<u>however</u>** she barely tolerates him.
(8) John loves Mary**<u>;</u>** she barely tolerates him.
(9) **<u>It is both true that</u>** John loves Mary **<u>as well as that</u>** she barely tolerates him.

Undoubtedly sentences (2)-(9) do differ in meaning, but they are all true if and only if both components are true, which is why they are all represented as [2].

In the above examples, the components of the conjunctions were simple propositions, but this need not be the case. Here is one example where both conjuncts are negations:

(10) Andrew has **no** job, **in addition,** he ca**n't** cook.

J: Andrew has a job
C: Andrew can cook

[10] ~J • ~C

In the case where one of the conjuncts is a conjunction we will need to enclose the conjunction-component in parentheses:

(11) Ann has a dog **while** Betty has **both** a dog **and** a cat.

A: Ann has a dog
D: Betty has a dog
C: Betty has a cat

[10] A • (D • C)

We will say more about the importance of parentheses in section 5.

Exercise "Conjunctions"

Symbolize the following statements using the symbolization key provided.

A: Abe will make dinner
B: Betty will make dinner
C: Chris will make lunch

(a)	Abe and Betty will both make dinner.	
(b)	Abe will make dinner though Betty will not.	
(c)	Abe will not make dinner even though Betty will.	
(d)	Abe and Betty will make dinner while Chris will make lunch.	
(e)	Chris will not make lunch; moreover, Abe will not make dinner	
(f)	Despite the fact that Abe will not make dinner, Betty will make it.	
(g)	Abe will not make dinner but still Betty will make it.	

4.3. Disjunction

Connective:	or
Symbol:	$\lor$ [the wedge]
Logical form:	$\square \lor \square$
	$p \lor q$
Components:	p, q – disjuncts

Here is an example of a disjunction:

(1) Ann has an apple **<u>or</u>** Ann has a banana.

Again we would rather say "Ann has an apple or a banana" or "Ann has either an apple or a banana", which expresses the very same proposition that sentence (1) expresses.

The components of a disjunction are called disjuncts. In (1), the first disjunct is the proposition "Ann has an apple", while "Ann has a banana" is the second disjunct.

The Basic Truth Table for Disjunction

Unlike the truth tables for negation and conjunction the truth table for disjunction is not as intuitive. There is an intuitive problem with the first row of the truth table. Let us consider the unproblematic rows first.

	p	q	$p \lor q$
[What Ann has:]	Ann has an apple	Ann has a banana	Ann has an apple or a banana
	T	F	

If it is true that Ann has an apple but it is not true that Ann has a banana, then it will be true to say that Ann has an apple or a banana. Likewise:

	p	q	$p \lor q$
[What Ann has:]	Ann has an apple	Ann has a banana	Ann has an apple or a banana
	F	T	

If it is not true that Ann has an apple but it is true that Ann has a banana, then it will be true to say that Ann has an apple or a banana.

[What Ann has:]	p Ann has an apple	q Ann has a banana	$p \vee q$ Ann has an apple or a banana
	F	F	

If it is neither true that Ann has an apple nor that Ann has a banana, then it will be false to say that Ann has an apple or a banana.

Now, consider the first row of the truth table, where it is both true that Ann has an apple and that Ann has a banana. Imagine that I say "Ann has an apple or a banana". Is what I have said true or false?

Well, when I ask this questions of students in a logic class, about 50% say that the sentence is true and another 50% say that the sentence is false, and there are some students who change their mind in the middle. This is because we sometimes use a disjunction in an *exclusive* manner and sometimes in an *inclusive* manner.

When a strict kitchen lady says " you can have either chocolate cake or ice-cream for dessert" while wagging her finger at you and raising her eye-brows as if to warn you, you will rather naturally understand what she says in an exclusive way, to mean that:

> you can have either chocolate cake or ice-cream but you cannot have both of them.

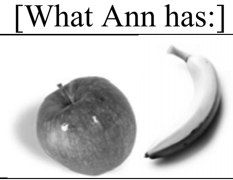
(The exclusive disjunction is false when both disjuncts are true.)

However, there are other situations when we understand disjunction in an inclusive manner. The sentence "John or Mary will work late" certainly leaves open the possibility that John as well as Mary might work late. Read most naturally it represents an inclusive disjunction:

> Either John will work late or Mary will work late or both John and Mary will work late.

The sentence "John or Mary will work late" simply says that *at least one* of them will work late.

Logicians have decided to understand the wedge '∨' as the symbol of *inclusive disjunction*, which is true when both disjuncts are true:

[What Ann has:]	p Ann has an apple	q Ann has a banana	$p \vee q$ Ann has an apple or a banana
	T	T	T

Sometimes, though rarely, a different symbol (⊥) is used to represent exclusive disjunction. But as you will see in the next unit, there is actually a sure-fire way of representing any exclusive disjunction by means of the inclusive disjunction, negation and conjunction. So we will not be losing anything.

I have said that it was a theoretical decision to treat the inclusive disjunction as in some ways primary. You should not take this to mean that it was an arbitrary decision. There are good reasons for it. For example, if we treat most occurrences of ‘or’ as representing inclusive disjunction, we will be able to capture some logical relations between disjunctions and other complex propositions. We will begin to study those relations soon. At the end of the course, the more curious among you might turn back here and ask yourselves whether, if the logicians adopted the exclusive disjunction as the primary disjunction, they would have been able to display some of the logical relations between disjunctions and other types of statements.

> A disjunction is true if and only if *any* disjunct is true. A disjunction is false if and only if both disjuncts are false.

Fill in the truth-values. (You should check at the end of this unit that you have done so correctly.)

Truth-value of disjunction:

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

Disjunctions in English

Disjunctions are not as abundant as conjunctions in English. While some are not so obvious (as you will see in the next unit), most are indicated by the occurrence of the following phrases:

... or ...
either ... or ...
… or else …

Consider some examples:

(2) Ann will marry Burt <u>**or**</u> Chris.

B: Ann will marry Burt
C: Ann will marry Chris

[2] B ∨ C

You might point out that the most natural interpretation of the sentence (2) would be as an exclusive disjunction. However, we will stick by the rule that unless the exclusive disjunction is indicated by the addition of a phrase like “but not both”, we will treat all disjunctions as inclusive disjunctions.

As before the components of disjunctions may complex:

(3) **<u>Either</u>** Susan will get a dog **<u>and</u>** a parrot **<u>or</u>** she will get a cat **<u>but</u>** **<u>not</u>** a parrot.

C: Susan will get a cat
D: Susan will get a dog
P: Susan will get a parrot

[3] (D • P) ∨ (C • ~P)

Exercise "Disjunctions"

Symbolize the following statements using the symbolization key provided.

A: Abe will make dinner
B: Betty will make dinner
C: Chris will make lunch
D: Dan will make lunch

(a) Abe or Betty will make dinner.

(b) Either Abe or Betty will make dinner.

(c) Chris will make lunch while either Abe or Betty will make dinner.

(d) Either Chris or Dan will make lunch.

(e) Either Chris will make lunch or Betty will not make dinner.

(f) Either Chris will not make lunch or Dan will not make lunch.

(g) Either Chris or Dan will make lunch and either Abe or Betty will make dinner.

(h) Abe and Betty will make dinner.

(i) Either Abe and Betty will make dinner or Chris and Dan will make lunch.

(j) Either Abe will make dinner while Chris will make lunch or Betty will make dinner while Dan will make lunch.

(k) Betty will make dinner and either Chris or Dan will make lunch.

(l) Either Abe will make dinner or either Chris or Dan will make lunch.

4.4. Material Biconditional

Connective:	if and only if
Symbol:	≡ [the triplebar]
Logical form:	□ ≡ □
	$p \equiv q$
Components:	*p*, *q* – terms of the biconditional

Here is an example of a biconditional (we will only be talking about material biconditionals, so we can skip the adjective 'material' in most discussions):

(1) Ann will be elected if and only if Ben will be elected.

In (1), the first term of the biconditional is the proposition "Ann will be elected", while "Benn will be elected" is the second term.

The Basic Truth Table for the Biconditional

The biconditional says, roughly, that one thing happens if and only if (in the same circumstances in which) the other thing does. Let us consider this on an example starting first with situations when the biconditional is evidently false.

	p	q	$p \equiv q$
[Who was elected:]	Ann is elected	Ben is elected	Ann will be elected if and only if Ben will be elected
	T	F	

Suppose that it is true that Ann was elected but Ben was not. It seems quite intuitive to claim that the prediction that "Ann will be elected if and only if Ben is elected" turned out false.

	p	q	$p \equiv q$
[Who was elected:]	Ann is elected	Ben is elected	Ann will be elected if and only if Ben will be elected
	F	T	

Likewise if Ben was elected while Ann was not, the prediction that "Ann will be elected if and only if Ben is elected" turned out false.

[Who was elected:]	p	q	$p \equiv q$
	Ann is elected	Ben is elected	Ann will be elected if and only if Ben will be elected
	T	T	

If both Ann and Ben were elected then the prediction that "Ann will be elected if and only if Ben is elected" would in fact be confirmed. Now:

[Who was elected:]	p	q	$p \equiv q$
	Ann is elected	Ben is elected	Ann will be elected if and only if Ben will be elected
	F	F	

If neither Ann nor Ben were elected then the prediction that "Ann will be elected if and only if Ben is elected" would also confirmed. For the statement that "Ann will be elected if and only if Ben is elected" is not a prediction that both of them *will* be elected rather it is a prediction that they will stand or fall together. In other words, it is a claim that either they will be both elected or they will both not be elected.

The biconditional is true if and only if its terms have the same truth-value. The biconditional is false if and only if its terms have different truth-values.

Fill in the truth-values. (You should check at the end of this unit that you have done so correctly.)

Truth-value of biconditional:	p	q	$p \equiv q$
	T	T	
	T	F	
	F	T	
	F	F	

Biconditionals in English

The biconditionals are indicated by the occurrence of the following phrases:

... if and only if ...
... if but only if...
… when and only when …
… just in case …
… iff …
… exactly if …

Biconditionals are often used in definitions, in which you frequently see the abbreviation 'iff', which is simply read as 'if and only if'.

Consider some examples of biconditionals:

(2) There is a thunder just in case there is a lightning.

T: There is a thunder
L: There is a lighting

[2] T ≡ L

Here is another example with more complex terms:

(3) You will get an A+ in this course just in case you receive 100% on all the quizzes and USM recognizes A+ as a grade.[1]

A: You get an A+ in this course
Q: You receive 100% on the quizzes
U: USM recognizes A+ as a grade

[3] A ≡ (Q • U)

Exercise "Biconditionals"

Symbolize the following statements using the symbolization key provided.

A: Abe will make dinner
B: Betty will make dinner
C: Chris will make lunch

(a) Abe will make dinner if and only if Betty will.

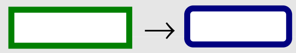
(b) Chris will make lunch just in case Betty makes dinner.

(c) Abe makes dinner when only when Betty does not.

(d) Chris will make lunch just in case Abe or Betty make dinner.

[1] Note that I have treated (3) as a proposition, though it is ambiguous because of the occurrence of 'you'. Given, however, that sentences with indexicals are easily disambiguated by context, I will from now on allow such sentences in examples, for they often work better on our intuitions.

4.5. Material Conditional

Connective:	if … then …
Symbol:	$\rightarrow$ [the arrow]
Logical form:	
	$p \rightarrow q$
Components:	p – antecedent q – consequent

Note that the material conditional is often symbolized by means of a horseshoe $\supset$, so it has the form $p \supset q$. However, we will be using an arrow because it is easier to get a look-alike in WebCT.

The conditional is the hardest among the connectives. It is hard to symbolize. It is hard to understand its truth table. But the basic idea behind a conditional is simple. A conditional says that something (which is expressed in the consequent) is true on some condition (which is expressed in the antecedent).

> If it rains then I will get wet.
> If you get more than 90 points, you will get an A.
> If Mary puts on the new dress, Newt will ask her out.

(Mark the antecedents in green and the consequents in blue in the above sentences.)

Conditionals in English

Let us consider the following conditional

(1) **<u>If</u>** Mary puts on the new dress **<u>then</u>** Newt will ask her out.

The antecedent is the proposition "Mary puts on the new dress". It expresses the condition on which the consequent ("Newt will ask Mary out") is true.

There are numerous phrases by means of which we can express conditionals in English:

... if ...
... provided that ...
... given that ...
... in case ...
... in the event that ...
... as long as ...
... assuming that ...
... supposing that ...
... on the condition that ...
... on the assumption that ...

One point of difficulty is that we often need to use our intuitions to help us determine what the antecedent and what the consequent is. There is no simple rule that would tell us that the first proposition is the antecedent while the second is the consequent. Let us formulate the proposition expressed by sentence (1) using the connective phrases just listed. (Mark the antecedent "Mary puts on the new dress" in green, and the consequent "Newt will ask Mary out" in blue.)

> **Assuming that** Mary puts on the new dress**,** Newt will ask her out.
> **On the condition that** Mary puts on the new dress**,** Newt will ask her out.
> **Provided that** Mary puts on the new dress**,** Newt will ask her out.
> **Given that** Mary puts on the new dress**,** Newt will ask her out.
> Newt will ask Mary out **if** she puts on the new dress.
> Newt will ask Mary out **in case** she puts on the new dress.
> Newt will ask Mary out **as long as** she puts on the new dress.
> Newt will ask Mary out **provided that** she puts on the new dress.
> Newt will ask Mary out **given that** she puts on the new dress.
> Newt will ask Mary out **on the condition that** she puts on the new dress.

It is of vital importance in the symbolization of each conditional to determine what the antecedent is and what the consequent is. It is also a good idea to paraphrase each conditional into a standard "if… then…" form before symbolizing it. Let us see this on a couple of examples.

(2) **Assuming that** you manage to buy a suit**,** we can marry even today**.**

The condition here is your managing to buy a suit – it is the condition of our marrying. Let's paraphrase (2) into a standard 'if… then…' form:

(2′) **If** you manage to buy a suit **then** we can marry even today**.**

(2′) is a good paraphrase of (2) when we intuitively understand both to mean the same thing (to have the same truth conditions), which is certainly the case here. We can thus symbolize (2) thus:

S: You manage to buy a suit
M: We can marry today.

[2] $S \rightarrow M$

Consider another sentence:

(3) The cat will stop scratching the furniture **if** you train her well.

Identify the antecedent (the condition) and the consequent and paraphrase:

(3′) **If** you train the cat well **then** she will stop scratching the furniture.

C: You train the cat well
S: The cat stops scratching furniture

[3] $C \rightarrow S$

Consider yet another example:

(4) Edward will even move to Siberia **on the condition that** Susan **and** Jane go with him.

Paraphrasing:

(4′) **If** Susan **and** Jane go with Edward **then** he will even move to Siberia.

E: Edward will move to Siberia
J: Jane goes with Edward
S: Susan goes with Edward

[4] $(S \bullet J) \rightarrow E$

Exercise "Conditionals 1"

Symbolize the following statements using the symbolization key provided.

A: Abe will make dinner
B: Betty will make dinner
C: Chris will make lunch
D: Dan will make lunch

(a) If Chris makes lunch then Betty will make dinner.

(b) Chris will make lunch if Dan does not make lunch.

If

then

(c) Dan will not make lunch if Abe does not make dinner.

If

then

(d) Dan will make lunch if either Abe or Betty make dinner.

If

then

(e) On the condition that Betty makes dinner, Chris will make lunch.

If

then

(f) Betty will make dinner on the condition that Chris makes lunch.

If

then

(g) Abe will make dinner on the assumption that Betty does not.

If

then

(h) Assuming that Chris does not make lunch, Dan will make it.

If

then

(i) Abe will make dinner provided that either Chris or Dan make lunch.

If

then

(j) Provided that Abe will make dinner, Betty will not make it.

If

then

(k) As long as Abe makes dinner, Betty will not make it.

If

then

(l) Given that either Chris or Dan make lunch, Abe or Betty will make dinner.

If

then

(m) Chris and Dan will both make lunch given that Abe and Betty both make dinner.

If

then

(n) It would be false to claim that if Betty makes dinner then Abe will make dinner.

(o) It would be false to claim that if Betty does not make dinner then Abe will not make dinner.

Exercise "Conditionals 2"

Please symbolize the following sentences, using the following symbolization key. Paraphrase the sentences where indicated into the "if… then…" form.

D: Ann is on a diet
E: Ann exercises regularly
F: Ann is getting fat
H: Ann is getting healthier
I: Billy is on a diet
J: Billy jogs regularly
T: Billy is getting fat

(a) If Ann exercises regularly, she will get healthier.

(b) Ann will get healthier if she goes on a diet.

If

then

(c) Ann will go on a diet if Billy goes on a diet

If

then

D: Ann is on a diet
E: Ann exercises regularly
F: Ann is getting fat
H: Ann is getting healthier
I: Billy is on a diet
J: Billy jogs regularly
T: Billy is getting fat

(d) Billy will go on a diet provided that Ann goes on a diet

If

then

(e) Given that Billy jogs regularly, he is not getting fat.

If

then

(f) Ann will exercise regularly on the condition that Billy jogs regularly.

If

then

(g) Provided that Ann is not getting fat, she will be getting healthier.

If

then

(h) Ann exercises regularly given that she is getting healthier.

If

then

(i) On the condition that Billy jogs regularly, Ann will be exercising regularly as well.

If

then

The Basic Truth Table for the Conditional

Let us turn to the truth table for the conditional. I should warn you at the outset that there is no intuitive interpretation for the way in which the truth table turns out in the last two rows. I will provide a theoretical explanation but intuitively you will (for good reasons) be unsatisfied.

Let us begin, however, with the only row in the truth table for the conditional that really *is* intuitive. Let us take an example of a conditional:

(1) If Snow White eats the apple then she will die.

When is the conditional false?

Let us try to think about the conditional "If Snow White eats the apple then she will die" as a kind of prediction that we are making. Suppose that we see that Snow White is about to eat an apple and we think that if she does, she will die. When would we be wrong?

	p	q	$p \rightarrow q$
	Snow White eats the apple	Snow White dies	If Snow White eats the apple then she will die.
			F

It seems rather intuitive to think that we would be wrong in our prediction, if Snow White ate the apple but did not die, i.e.:

	p	q	$p \rightarrow q$
	Snow White eats the apple	Snow White dies	If Snow White eats the apple then she will die.
	T	F	F

Consider another example.

(2) It you stir cornstarch in water, it will dissolve.

We know that (2) is false because when you make the antecedent true (when you put cornstarch into water and stir it), you do not thereby make the consequent true (it will not dissolve).

This gives us the intuitive part of the truth table for the conditional.

p	q	$p \rightarrow q$
T	T	
T	F	F
F	T	
F	F	

When is the conditional true?

The short answer is: in all remaining cases, i.e. when it is not false.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

“But why?”, you will ask.

Well, we can understand why it is true in the first row. Our prediction about the Snow White will turn out to be true in the situation in which the Snow White does eat the apple and does die:

	p	q	$p \rightarrow q$
	Snow White eats the apple	Snow White dies	If Snow White eats the apple then she will die.
 	T	T	T

But there are no intuitive reasons why the conditional should be true in cases when the antecedent is false. It must seem absurd to you that it should be so, but this is the way it is. I say so, and you must trust me ☺.

This is in fact a point at which it becomes apparent that propositional theory, though it is a good theory in a great many ways, is simply too weak to capture our intuitions about conditionals.

Although the theory does not have a way of accommodating all of our intuitions about conditionals, there are good theoretical reasons why the truth table for the conditional must, in propositional logic, look the way it does. One of these reasons is mentioned in the box that follows, which you might read later if you are interested. The other reason is that given this choice of the truth table for the conditional, we can make sense of numerous logical relations between conditionals and other propositions. For example, we will be able to understand that a biconditional $p \equiv q$ is logically equivalent to a conjunction of conditionals $(p \rightarrow q) \bullet (q \rightarrow p)$.

A conditional is false just in case its antecedent is true and its consequent false. A conditional is true just in case either its antecedent is false or its consequent is true.

You must learn the truth table for the conditional by heart!

Fill in the truth-values. (You should check at the end of this unit that you have done so correctly.)

Truth-value of conditional:

p	q	$p \to q$
T	T	
T	F	
F	T	
F	F	

Why is the conditional true when the antecedent is false?

The truth table for the conditional is intuitive in its first "half":

p	q	$p \to q$
T	T	1
T	F	0
F	T	
F	F	

The question that we might ask is what the truth-values for the other "half" should be. One way to approach the question is to consider all the possibilities of filling the missing truth-values in the last two rows. There are exactly four such possibilities:

p	q	p ❶ q
T	T	1
T	F	0
F	T	1
F	F	1

p	q	p ❷ q
T	T	1
T	F	0
F	T	1
F	F	0

p	q	p ❸ q
T	T	1
T	F	0
F	T	0
F	F	1

p	q	p ❹ q
T	T	1
T	F	0
F	T	0
F	F	0

It turns out that only truth table for the connective marked as ❶ is not already "used up." Let us consider the connectives in reverse order:

The truth table for ❹ is nothing else than the truth table for the conjunction.

The truth table for ❸ is nothing else than the truth table for the biconditional.

The truth table for ❷ is nothing else than the truth table for consequent q. But it would be quite disastrous to claim that the truth conditions for a conditional are the same as the truth conditions for the consequent of the conditional. Intuitively, the truth of the conditional depends not only on the consequent but also on the antecedent. If ❷ were accepted as the meaning of the conditional then the statement "If you owe me one million dollars, then you should pay me back at least that much" would mean the same as "You should pay me at least one million dollars". But this would be absurd.

And thus we are left with the truth table for ❶ as the only available option in propositional logic

Source: V. Klenk, *Understanding Symbolic Logic* (Upper Saddle River, NJ: Prentice Hall, 2002)

5. Main Connective

In this last section, you will learn what the main connective is. Very roughly, the main connective is the primary or the most important or the dominating connective in a proposition. We will proceed in the following way. First, we will see why the idea of the main connective is so important (§5.1). Then we will learn a method for determining what the main connective of a proposition is. We will do this in two stages to make it more perspicuous: for the two-place connectives only (§5.2) and then for all the connectives including the one-place connective of negation (§5.3).

5.1. When It Is Unclear What the Main Connective Is…

Example 1.

Consider the following utterance – Bubba says to Kate:

I will show you my stamps or make you coffee and give you $1000.

Bubba's utterance is ambiguous. It can be understood in two *radically* different ways:

Interpretation A (optimistic)

(I will show you my stamps or make you coffee) and give you $1000.

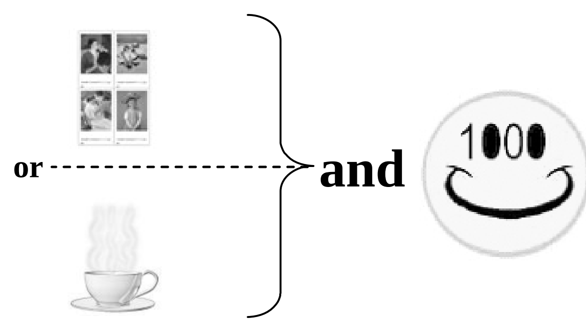
C: Bubba will make Kate some coffee
S: Bubba will show Kate stamps
T: Bubba will give Kate $1000

[1] $(S \vee C) \bullet T$

In other words,

(1) Bubba will show Kate stamps or make her coffee but, in any event, he will give her $1000.

The main connective in this proposition is the dot, which means that the whole proposition is a conjunction. Here is a way of representing what the proposition says:



The important point is that Kate will receive $1000 whether she is shown stamps or given coffee. You can see why, for Kate, this is a very optimistic interpretation of Bubba's utterance.

Interpretation B (realistic)

I will show you my stamps or (make you coffee and give you $1000).

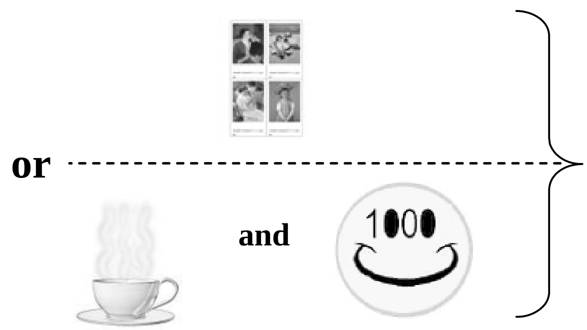
C: Bubba will make Kate some coffee
S: Bubba will show Kate stamps
T: Bubba will give Kate $1000

[2] S ∨ (C • T)

In other words,

(2) Either Bubba will show Kate stamps or he will both make her coffee and give her $1000.

The main connective in this proposition is the wedge, which means that the whole proposition is a disjunction. Here is a way of representing what the proposition says:



On this interpretation, if we treat the proposition as a disjunction, Bubba tells Kate that he will do one of two things: either he will show her stamps, or he will both make her coffee and give her $1000.

You can now appreciate the difference between [1] and [2]. If what Bubba says is [1] then he is committed to giving Kate $1000. But if what he says is [2], then he does not commit himself to giving Kate $1000 – he has the option of just showing her the stamps and his promise would still be kept.

From the semantic point of view (from the point of view of what the statements mean), the difference between them is humongous, as we have just seen. But from a syntactic point of view (from the point of view of the logical structure of the statements), the difference has to do with the way the parentheses are placed (fill in the blanks):

[1] (S ∨ C) • T main connective:
[2] S ∨ (C • T) main connective:

This is only to say that the way the parentheses are placed is of crucial importance. For two-place connectives, the location of the parentheses determines what the main connective of a statement is.

You might be wondering whether Bubba's original utterance should not be represented as S ∨ C • T… Well, after you have thought this thought, quickly erase it! The above sequence of symbols is logical gibberish – it does not represent any proposition (remember that propositions *cannot* be ambiguous).

Example 2

Let’s consider one more example of an ambiguous utterance – Bubba says:

I will drink coffee and read a newspaper or go to work.

C: Bubba will drink coffee
N: Bubba will read a newspaper
W: Bubba will go to work

Interpretation A

(I will drink coffee and read a newspaper) or go to work.
Either

or

$(C \bullet N) \vee W$

Main connective:

Interpretation B

I will drink coffee and (read a newspaper or go to work).
and either

or

$C \bullet (N \vee W)$

Main connective:

5.2. Finding the Main Connective – Stage 1

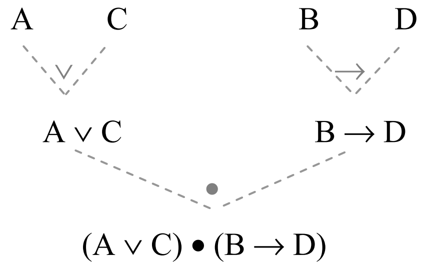
5.2.1. Main Connective in Proposition Construction

The main connective determines what kind of proposition a given proposition is: whether it is a negation, a conjunction, a disjunction, a biconditional or a conditional. Fill in the following table (if you are unsure, read on for pointers):

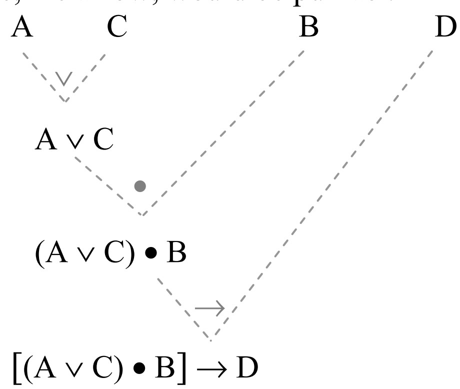
	main connective	proposition type
$(A \vee C) \bullet (B \rightarrow D)$	$\bullet$ (the dot)	conjunction
$A \vee [C \bullet (B \rightarrow D)]$		
$[(A \vee C) \bullet B] \rightarrow D$		

$A \vee [C \bullet (B \rightarrow D)]$ is a disjunction, its main connective is the connective of disjunction (the wedge); $[(A \vee C) \bullet B] \rightarrow D$ is a conditional; its main connective is the arrow.

If we were to imagine the process of constructing a complex proposition out of simple propositions then the main connective is the last connective used in constructing the proposition. We could imagine the construction of proposition (A ∨ C) • (B → D) in the following way:



The proposition [(A ∨ C) • B] → D would be constructed in such a way that the conditional connective, the arrow, would be put last:



Reconstruct the way in which proposition A ∨ [C • (B → D)] would be constructed:

A C B D

A ∨ [C • (B → D)]

5.2.2. The Method of Binding Parentheses

It is important for you to be able to determine what the main connective is in an arbitrarily complex proposition, even such as this:

$$(((A \vee B) \bullet (C \vee D)) \rightarrow C) \equiv (A \rightarrow ((C \rightarrow D) \rightarrow B))$$

Fortunately there is a simple method that helps to make this task manageable. It consists in binding parentheses in pairs. Let us begin with a simpler example, however.

Example 1

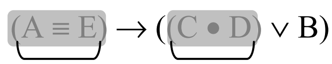
What is the main connective of the following proposition?

$$(A \equiv E) \rightarrow ((C \bullet D) \vee B)$$

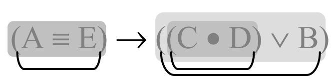
We begin by binding (or pairing) the innermost parentheses that connect the simple propositions:

$$\underbrace{(A \equiv E)} \rightarrow (\underbrace{(C \bullet D)} \vee B)$$

The propositions thus marked we treat (in thought) as wholes (whole propositions):



We repeat the process binding the next level of parentheses and treating the propositions thus marked as whole:



It becomes clear that the proposition is a conditional, the main connective is the arrow:

Example 2

What is the main connective of the following proposition?

$$(((A \vee B) \bullet (C \vee D)) \rightarrow C) \equiv (A \rightarrow ((C \rightarrow D) \rightarrow B))$$

Try to do this on your own and check that you have done it correctly:

We begin by binding (or pairing) the innermost parentheses that connect the simple propositions, treating thus marked propositions as wholes:

$$((A \vee B) \bullet (C \vee D)) \rightarrow C) \equiv (A \rightarrow ((C \rightarrow D) \rightarrow B))$$

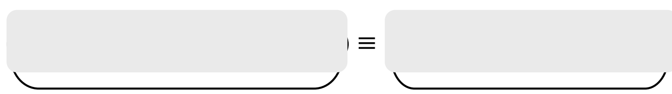
We repeat the process binding the next level of parentheses and treating the propositions thus marked as whole:

$$((A \vee B) \bullet (C \vee D)) \rightarrow C) \equiv (A \rightarrow ((C \rightarrow D) \rightarrow B))$$

And so on, until we have bound all the parentheses:

$$((A \vee B) \bullet (C \vee D)) \rightarrow C) \equiv (A \rightarrow ((C \rightarrow D) \rightarrow B))$$

You can see now what the main connective is:



The main connective is the triplebar – the proposition is a biconditional.

Exercise "Main Connective 1"

Using the method of parentheses binding, find the main connective in each of the following propositions:

1. $(A \bullet B) \vee (C \rightarrow D)$
2. $A \bullet (B \vee (C \rightarrow D))$
3. $((A \bullet B) \vee C) \rightarrow D$
4. $A \bullet ((B \vee C) \rightarrow D)$
5. $(A \bullet (B \vee C)) \rightarrow D$
6. $(A \rightarrow A) \rightarrow (A \rightarrow B)$
7. $A \rightarrow (A \rightarrow (A \rightarrow B))$
8. $((A \rightarrow A) \rightarrow A) \rightarrow B$
9. $(((A \bullet B) \bullet C) \equiv (A \vee C)) \rightarrow (A \bullet (B \vee C))$
10. $(((A \equiv B) \rightarrow (B \equiv C)) \bullet (C \rightarrow D)) \vee (B \rightarrow ((A \bullet B) \equiv C))$
11. $(((A \vee B) \bullet (C \vee D)) \rightarrow C) \equiv ((A \rightarrow ((C \rightarrow D) \rightarrow B)) \rightarrow D)$

5.3. Finding the Main Connective – Stage 2

So far we have only considered two-place connectives. We must now add the one-place connective of negation.

5.3.1. Two Examples

Let us consider two propositions:

[1] ~(V • L)

[2] ~V • L

The proposition [1] is the *negation* of the conjunction V • L – the main connective of proposition [1] is the connective of negation, the tilde. The proposition [2] is a *conjunction* whose first conjunct is the negation ~V – the main connective is thus the connective of conjunction, the dot.

The difference between these sentences is again enormous. Consider the following symbolization key:

V: Adam will go to heaven
L: Adam will go to hell

(1) It is not the case that: Adam will go both to heaven and to hell.

(2) Adam will go not to heaven but to hell.

Proposition (1) says something that is certainly true – nobody can go to heaven and hell at the same time, so Adam will not go to both places. But proposition (1) does not tell us *where* Adam will go. Proposition (2), on the other hand, tell us exactly where he will go, though we know that he won't be happy about it.

Consider another pair of propositions:

[3] ~(A • C)

[4] ~A • C

Write in the propositions using the following symbolization key:

A: Ben will get an A in logic
C: Ben will get a C in logic

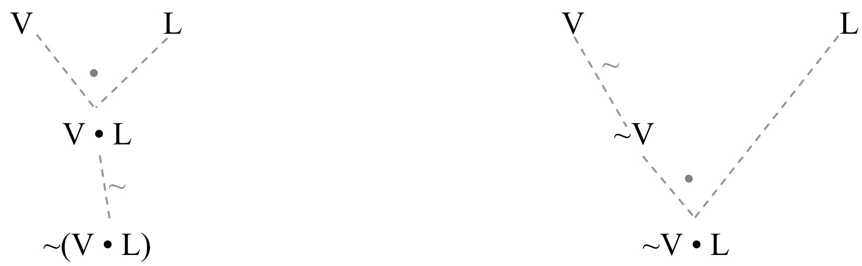
(3)

(4)

Proposition (3) again says something true: Ben will not get both an A and a C in logic, but it does not tell us what he will get. Proposition (4) tells us exactly what Ben will get: not an A, but a C.

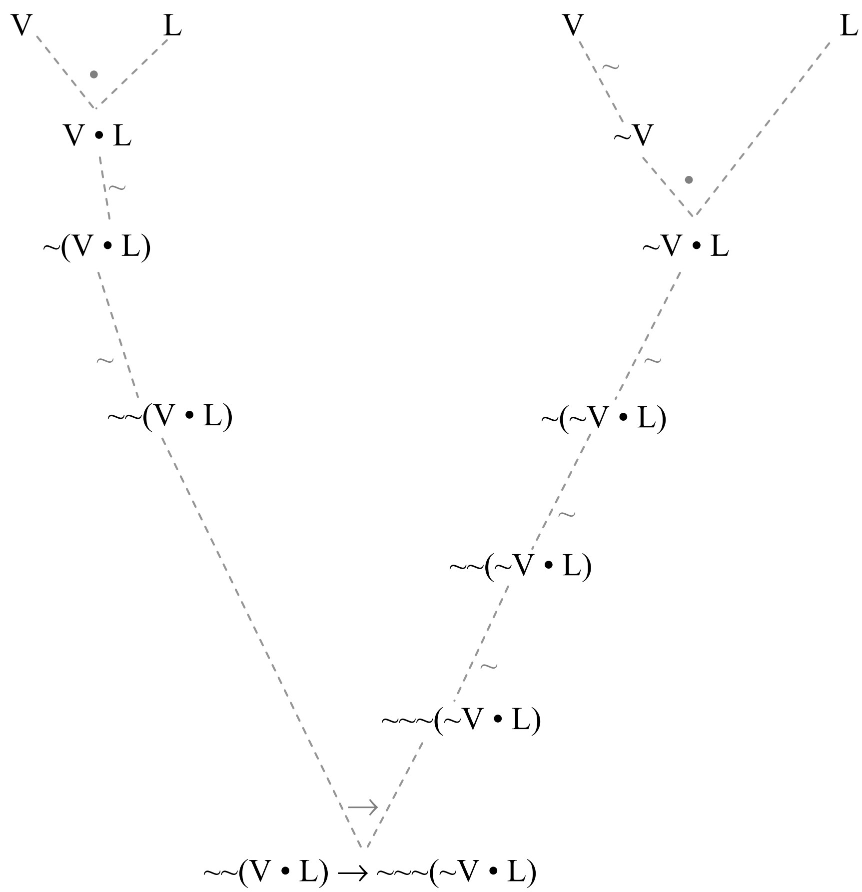
5.3.2. Main Connective in Proposition Construction (Including Negation)

If we were to imagine the process of constructing the propositions we were just talking about, it would consist in combining negation and conjunction but in reverse order.



In proposition ~(V • L), the tilde is put in last and it is the main connective. In proposition ~V • L, the dot is put in last and it is therefore the main connective.

Of course we can construct more complex propositions. Here is the representation of the construction of proposition ~~(V • L) → ~~~(~V • L):



5.3.3. The Method of Binding Parentheses (Including Negation)

The connective of negation differs from the other connectives in that it is a one-place connective. Because it is a one-place connective, we adopt the convention of not marking complex negations with parentheses, though we could do so. Here is a contrast between following the convention where all complex propositions (including negations) are marked by parentheses, let us call it convention "They", and our convention "We". Here are some examples:

"They"	"We"
~(~B)	~~B
~(~(~C))	~~~C
(~A) ∨ (~B)	~A ∨ ~B
~(~(~C)) ∨ (~B)	~~~C ∨ ~B
(~(~B) • ~(~(~A))) → (~D)	(~~B • ~~~A) → ~D
~(~(A • B))	~~(A • B)
(~A) • B	~A • B
~(A • B)	~(A • B)

I hope that you agree that our convention is more perspicuous. (Note that the propositions in the last row will be represented in the same way in both conventions.)

Now, the cost of accepting our convention is that we have to mentally insert the parentheses into the proposition when we are determining what the main connective is.

Another way to put this is this: We have to remember is that the negation operator, the tilde, always binds what immediately follows it. Only three things can follow a tilde: a letter, a parenthesis and another tilde.

1) What follows '~' is a letter

In such a case what is negated is the simple proposition symbolized by the letter. In the proposition:

~A ∨ B

the '~' is followed by the letter 'A', which means that the negated proposition is the proposition A.

2) What follows '~' is an open parenthesis '('

In this case what is negated is the proposition that is enclosed by the parentheses. In the propositions

~(A ∨ B)
~(A ∨ B) → C

the '~' is followed by a parenthesis and each case what is negated is the proposition in those parentheses – in our case the negated proposition is the disjunction A ∨ B. In the proposition

~(A ∨ (B • C))

the negated proposition is the whole disjunction A ∨ (B • C).

3) What follows '~' is another '~'

In such a case what is negated by the first '~' is a negation. In the proposition

~~A ∨ B

the first tilde negates the proposition ~A, which itself is a negation. The second tilde of course negates the proposition A:

~~A ∨ B

Armed with this knowledge of the behavior of the '~' we can proceed to apply our method *tout court.* Let us apply the method to two example in parallel.

~(~A ∨ ~~B) • ~C   ~(~A ∨ ~(~B • ~C))

Again, we proceed from within, binding all tildes to letters first:

~(~A ∨ ~~B) • ~C   ~(~A ∨ ~(~B • ~C))

Then we bind the next level of propositions, thinking of the propositions that we already bound as whole:

~(~A ∨ ~~B) • ~C   ~(~A ∨ ~(~B • ~C))

We repeat this for the next level of propositions, thinking of the ones we have identified as wholes:

~(~A ∨ ~~B) • ~C   ~(~A ∨ ~(~B • ~C))

One final step is required:

~(~A ∨ ~~B) • ~C   ~(~A ∨ ~(~B • ~C))

to see that the first proposition is a conjunction while the second is a negation.

Exercise "Main Connective 2"

Using the method of parentheses binding, find the main connective in each of the following propositions:

1. $\sim A \rightarrow (B \vee A)$
2. $\sim(A \rightarrow B) \vee A$
3. $\sim((A \rightarrow B) \vee A)$
4. $\sim\sim A \vee (B \rightarrow C)$
5. $\sim(\sim A \vee B) \rightarrow C$
6. $\sim(\sim A \vee (B \rightarrow C))$
7. $\sim\sim(A \vee (B \rightarrow C))$
8. $\sim\sim(A \vee B) \rightarrow C$
9. $\sim(\sim(\sim A \vee \sim B) \rightarrow \sim(A \bullet B)) \rightarrow \sim(\sim A \vee \sim A)$
10. $\sim(\sim(A \rightarrow (B \bullet A)) \equiv \sim\sim\sim(\sim B \rightarrow \sim A))$
11. $\sim\sim(\sim A \vee \sim(B \bullet \sim C)) \rightarrow \sim((\sim C \vee B) \equiv A)$

6. Summary

We have introduced the basics of the propositional logic. In particular we have distinguished between simple propositions, which are symbolized by means of so-called propositional constants: A, B, C, etc, and complex propositions of which we have distinguished five types: negations, conjunctions, disjunctions, conditionals and biconditionals. We have learned that the main connective determines the kind of proposition a given proposition is. We have introduced some of the truth-functional semantics for propositional connectives, i.e. the basic truth tables.

p	$\sim p$
T	F
F	T

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Note! You should understand the basic truth tables and, then, memorize them!

And we have also began to learn to symbolize complex propositions, which we will continue to do in the next unit.

What You Need to Know and Do

- You need to understand the distinction between simple and complex propositions in propositional logic
- You need to be able to recognize the simple and complex propositions in practice
- You need to know the five connectives
- You need to know the terminology highlighted, what the types of propositions are called (negation; conditional, etc.), what the components of those propositions are called (the negated proposition; antecedent, consequent, etc).
- You need to know the English phrases that correspond to each of the connectives
- You need to know the truth tables for each of the connectives
- You need to be able to formulate the truth and falsehood conditions for each of the connectives. (For example, you need to know that the conditional is true just in case: the antecedent is false or the consequent is true.)
- You need to be able to do uncomplicated symbolizations.
- You need to understand what the main connective is.
- You need to be able to identify the main connective for an arbitrarily complex proposition.

Quiz Instructions

The quiz will consist of some general questions and some questions where you will be solving problems: deciding what the main connective is, doing symbolizations.

There may be multiple choice questions here (of both kinds) and open paragraph questions. See the Sample Quiz.

Symbolization Multiple-Choice Questions. Although most of the symbolization questions will be multiple-choice questions, it is far better to do the symbolization on a piece of paper before selecting the answer.

Note that the Sample quiz is only representative of the *kinds* of questions you can expect. It is not representative either of their difficulty or of the number of questions you can expect.

Note that there is no way to preserve the shape of the connective symbols in WebCT. We will be using the following symbols as WebCT stand-ins:

Proposition type	In symbols	WebCT substitute
Negation	$\sim p$	~p
Conjunction	$p \bullet q$	p . q
Disjunction	$p \vee q$	p v q
Biconditional	$p \equiv q$	p = q
Conditional	$p \rightarrow q$	p -> q
		Note that the arrow is composed of two characters: - and >

Symbolization Open Questions. You will be given an English sentence like:

Mary and Jane both have a dog.

Your task is twofold. First, you need to provide a symbolization key (see §3) and, second, you need to prove a symbolization given your symbolization key. Note that it really does not matter what letters you assign to the simple sentences, though it would be most natural to construct the following symbolization key:

J := Jane has a dog
M := Mary has a dog

in which case your symbolization ought to be:

M • J

Note that I will deduct points if you change the order of the simple propositions. So J • M is not a symbolization of the above sentence, though it is a symbolization of a logically equivalent sentence. However, if you decide to use the following symbolization key:

D := Jane has a dog
M := Mary has a dog

then your symbolization should be:

M • D

The quiz will be graded *manually*. Please be patient.
The quizzes should be graded on the next day after their deadlines.
I will post a note on the Bulletin Board
to let you know when they have been graded.

Further Reading

You can read about these matters further in a number of logic textbooks. I enclose the chapters titles for the textbooks I have chosen as optional.

Klenk: Ch. 2. The Structure of Symbolic Logic, Ch. 3.1. The Truth Tables for the Operators.

Hurley: Ch. 6.1 Symbols and Translation, Ch. 6.2. Truth Functions

Copi & Cohen: Ch. 8.1. The Symbolic Language of Modern Logic, Ch. 8.2. The Symbols for Conjunction, Negation and Disjunction, Ch. 8.3 Conditional Statements and Material Implication, Ch. 8.5.C. Material Equivalence